Chapter 5

Linear Inequalities and Linear Programming

Section 2

Systems of Linear Inequalities in Two Variables

Solving Systems of Linear Inequalities Graphically

- We now consider **systems** of linear inequalities such as

$$x + y > 6$$
$$2x - y > 0$$

- We wish to **solve** such systems **graphically**, that is, to find the graph of all ordered pairs of real numbers (x, y) that simultaneously satisfy all the inequalities in the system.
- The graph is called the **solution region** for the system (or **feasible region.)**
- To find the solution region, we graph each inequality in the system and then take the intersection of all the graphs.

Graphing a System of Linear Inequalities: Example

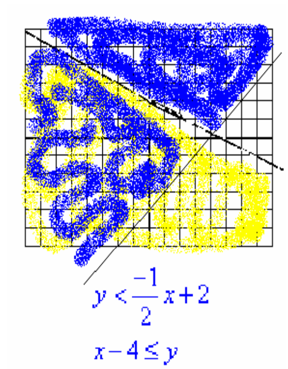
To graph a system of linear inequalities such as

$$y < \frac{-1}{2}x + 2$$
$$x - 4 \le y$$

we proceed as follows:

Graph each inequality on the same axes. The solution is the set of points whose coordinates satisfy all the inequalities of the system. In other words, the solution is the **intersection** of the regions determined by each separate inequality.

Graph of Example



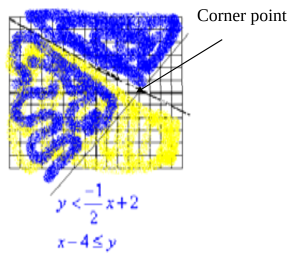
The graph of the first inequality $y < -(1/2)x + 2$ consists of the region shaded yellow. It lies below the **dotted line** $y = -(1/2)x + 2$.

The graph of the second inequality is the blue shaded region is above the **solid line** $x - 4 = y$.

The graph is the region which is colored **both blue and yellow**.

Corner Points

A **corner point** of a solution region is a point in the solution region that is the intersection of two boundary lines. In the previous example, the solution region had a corner point of (4,0) because that was the intersection of the lines *y* = –1/2 *x* + 2 and *y* = *x* – 4.



Bounded and Unbounded Solution Regions

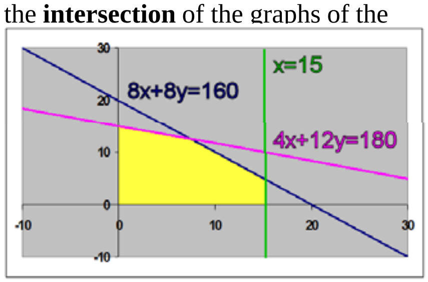
A solution region of a system of linear inequalities is **bounded** if it can be enclosed within a circle. If it cannot be enclosed within a circle, it is **unbounded**. The previous example had an unbounded solution region because it extended infinitely far to the left (and up and down.) We will now see an example of a bounded solution region.

Graph of More Than Two Linear Inequalities

To graph more than two linear inequalities, the same procedure is used. Graph each inequality separately. The graph of a system of linear inequalities is the area that is common to all graphs, or the **intersection** of the graphs of the individual in s.

Example:

$$x \ge 0$$
$$y \ge 0$$
$$x \le 15$$
$$8x + 8y \le 160$$
$$4x + 12y \le 180$$



Application

Suppose a manufacturer makes two types of skis: a trick ski and a slalom ski. Suppose each trick ski requires 8 hours of design work and 4 hours of finishing. Each slalom ski requires 8 hours of design and 12 hours of finishing. Furthermore, the total number of hours allocated for design work is 160, and the total available hours for finishing work is 180 hours. Finally, the number of trick skis produced must be less than or equal to 15.

How many trick skis and how many slalom skis can be made under these conditions? How many possible answers? Construct a set of linear inequalities that can be used for this problem.

Application
Solution

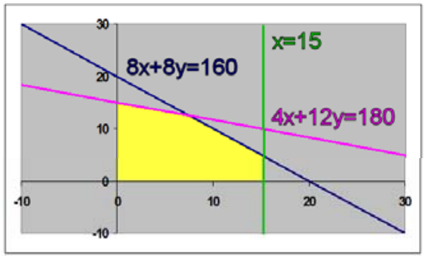
Let x represent the number of trick skis and y represent the number of slalom skis. Then the following system of linear inequalities describes our problem mathematically. Actually, only whole numbers for x and y should be used, but we will assume, for the moment that x and y can be any positive real number.

$$x \geq 0$$
$$y \geq 0$$
$$x \leq 15$$
$$8x + 8y \leq 160$$
$$4x + 12y \leq 180$$

- $x \geq 0$, $y \geq 0$: *x* and *y* must both be positive
- $x \leq 15$: Number of trick skis has to be less than or equal to 15
- $8x + 8y \leq 160$: Constraint on the total number of design hours
- $4x + 12y \leq 180$: Constraint on the number of finishing hours

Application
Graph of Solution

The origin satisfies all the inequalities, so for each of the lines we use the side that includes the origin.



The intersection of all graphs is the yellow shaded region.

The solution region is bounded and the corner points are (0,15), (7.5, 12.5), (15, 5), and (15, 0)