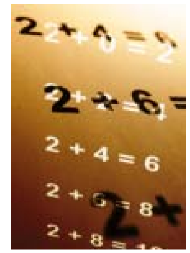
Chapter 5

Linear Inequalities and Linear Programming

Section 1

Linear Inequalities in Two Variables

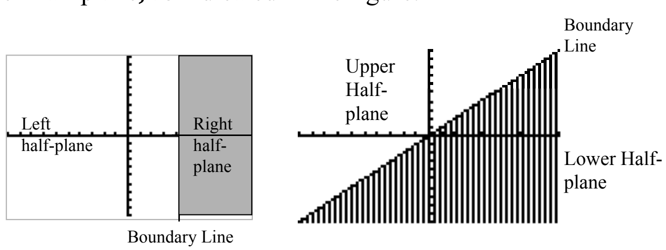
Systems of Linear Inequalities in Two Variables



In this section, we will learn how to graph linear inequalities in two variables and then apply this procedure to practical application problems.

Half-Planes

- A line divides the plane into two regions called **half planes**.
- A vertical line divides it into **left** and **right half planes**.
- A nonvertical line divides it into **upper** and **lower half planes.**
- In either case, the dividing line is called the **boundary line** of each half plane**,** as indicated in the figure.



Graphs of Linear Inequalities

- The graph of the linear inequality
  $Ax + By < C$
  or $Ax + By > C$
  with $B \neq 0$ is either the upper half-plane or the lower half-plane (but not both) determined by the line
  $Ax + By = C$.
- If $B = 0$ and $A \neq 0$, the graph of
  $Ax < C$
  or $Ax > C$
  is either the left half-plane or the right half-plane (but not both) determined by the line
  $Ax = C$.

Procedure for Graphing Linear Inequalities

Step 1. First graph $Ax + By = C$ as a dashed line if equality is not included in the original statement, or as a solid line if equality is included.

Step 2. Choose a test point anywhere in the plane not on the line (the origin (0,0) usually requires the least computation) and substitute the coordinates into the inequality.

Step 3. Does the test point satisfy the original inequality? If so, shade the half-plane that contains the test point. If not, shade the opposite half-plane.

Graphing a Linear Inequality Example 1

Our first example is to graph the linear equality

$$y < \frac{3}{4}x - 1$$

Graphing a Linear Inequality Example 1

Our first example is to graph the linear equality

$$y < \frac{3}{4}x - 1$$

Solution:

1. Replace the inequality symbol with an equal sign

$$y = \frac{3}{4}x - 1$$

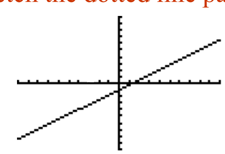
2. Graph the line.
   If the original inequality is a > or < sign, the graph of the line should be dotted, otherwise solid.

Example 1 (continued)

In this example, since the original problem contained the inequality symbol (<) the line that is graphed should be dotted.

For our problem, the equation of our line $y = \frac{3}{4}x - 1$ is already

in slope-intercept form, ($y = mx + b$) so we easily sketch the line by first starting at the y intercept of –1, then moving up 3 units and to the right 4 units, corresponding to our slope of ¾. After locating the second point, we sketch the dotted line passing through these two points.



Example 1
(continued)

3. Now, we have to decide which half plane to shade. The solution set will either be
   (a) the half plane above the line, or
   (b) the half plane below the graph of the line.

To determine which half-plane to shade, we choose a **test point** that is not on the line. Usually, a good test point to pick is the origin (0,0), unless the origin happens to lie on the line. In our case we can choose the origin as a test point.

Substituting the origin in the inequality $y < \frac{3}{4}x - 1$

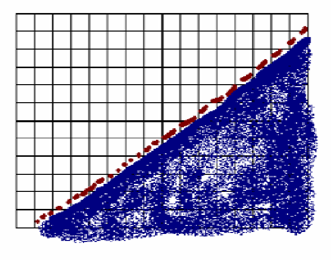
produces the statement $0 < 0 - 1$, or $0 < -1$.

Example 1
Graph

Since this is a **false** statement, we shade the region on the side of the line **not** containing the origin.

Had the origin satisfied the inequality, we would have shaded the region on the side of the line containing the origin.

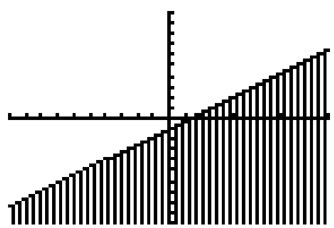
Here is the complete graph of the first inequality:



$$y < \frac{3}{4}x - 1$$

Example 1
Calculator Graph

We can also draw the graph on a graphing calculator, but we won't be able to graph the dotted boundary line.



Example 2

For our second example, we will graph the inequality $3x - 5y \geq 15$.

Example 2

For our second example, we will graph the inequality $3x - 5y \geq 15$.

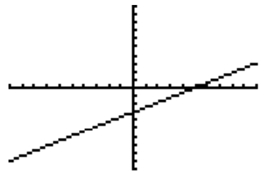
Step 1. Replace inequality symbol with equal sign:

$3x - 5y = 15$

Step 2. Graph the line $3x - 5y = 15$.

We will graph the line using the x and y intercepts: When $x = 0$, $y = -3$ and when $y = 0$, $x = 5$.

Plot these points and draw a **solid line.** The original inequality symbol is $\geq$, which means that the graph of the line itself is included. Graph is as shown.



Example 2 (continued)

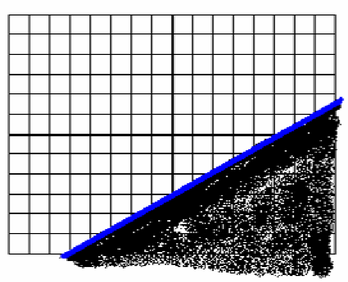
Step 3. Choose a point not on the line. Again, the origin is a good test point since it is not part of the line itself. We have the following statement which is clearly **false**.

$$3(0) - 5(0) \geq 15$$

Therefore, we shade the region on the side of the line that does **not** include the origin.

Example 2 (continued)

$$3x - 5y \geq 15$$



Example 3

Our third example is unusual in that there is no y variable present. The inequality $2x > 8$ is equivalent to the inequality $x > 4$. How shall we proceed to graph this inequality?

Example 3

Our third example is unusual in that there is no y variable present. The inequality $2x > 8$ is equivalent to the inequality $x > 4$. How shall we proceed to graph this inequality? The answer is: the same way we graphed previous inequalities:

Step 1: Replace the inequality symbol with an equals sign:

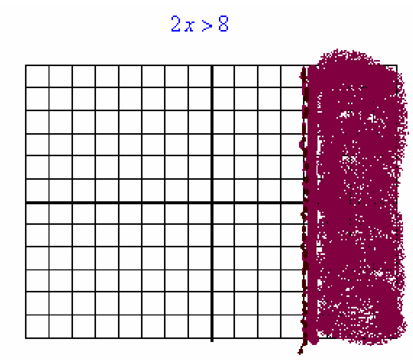
$x = 4$.

Step 2: Graph the line $x = 4$. Is the line solid or dotted? The original inequality is >. Therefore, the line is **dotted.**

Step 3. Choose the origin as a test point. Is $2(0) > 8$? Clearly **not**.

Shade the side of the line that does **not** include the origin. The graph is displayed on the next slide.

Example 3 Graph



Example 4

Graph $y \le -2$.

This example illustrates the type of problem in which the x variable is missing.

Example 4 Solution

Graph $y \le -2$.

This example illustrates the type of problem in which the x variable is missing. We will proceed the same way.

Step 1. Replace the inequality symbol with an equal sign:

$y = -2$

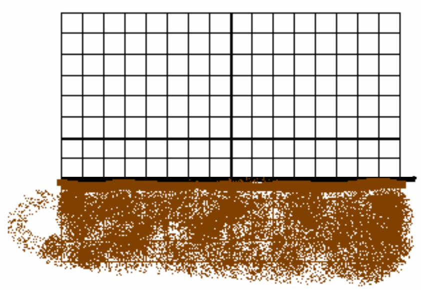
Step 2. Graph the equation $y = -2$. The line is solid since the original inequality symbol is $\le$.

Step 3. Shade the appropriate region. Choosing again the origin as the test point, we find that $0 \le -2$ is a **false** statement so we shade the side of the line that **does not** include the origin.

Graph is shown in next slide.

Example 4
Graph

y ≤ Š2



Application

A concert promoter wants to book a rock group for a stadium concert. A ticket for admission to the stadium playing field will cost \$125, and a ticket for a seat in the stands will cost \$175. The group wants to be guaranteed total ticket sales of at least \$700,000. How many tickets of each type must be sold to satisfy the group's guarantee? Express the answer as a linear inequality and draw its graph.

Application Solution

x = number of tickets sold for the playing field

y = number of tickets sold for seats in the stands

Total tickets sale must be at least \$700,000.

$$125x + 175y \geq 700{,}000$$

1. Graph: $5x + 7y \geq 28{,}000$
2. Test point (0, 0) and it's false.
3. The graph is the upper half-plane including the boundary line.

Application Solution

It's not possible to sell a negative number of tickets, we must restrict both x and y to the first quadrant.