**ODE Homework 1**

1. Convert each linear equation into a system of first order equations.
2. y″ − 4y′ + 5y = 0 2
3. y″′ − 5y″ + 9y = t cos 2t
4. y (4) + 3y″′ − πy″ + 2πy′ − 6y = 11 4

1. Rewrite the system you found in (a) Exercise 1 (a) and (b) Exercise 1 (b), into a matrix-vector equation X’ = A X
2. Convert the third order linear equation below into a system of 3 first order equation using (a) the usual substitutions, and (b) substitutions in the reverse order: x1 = y″, x2 = y′, x3 = y. Deduce the fact that there are multiple ways to rewrite each n-th order linear equation into a linear system of n equations.

y″′ + 6y″ + y′ − 2y = 0

1. Find the Eigenvalues and Eigenvectors for the 2 x 2 matrices manually (you could of course use Mathematica to check your answers but you should be able to do this manually as well)
2. $\left(\begin{matrix}0&1\\-5&4\end{matrix}\right)$
3. $\left(\begin{matrix}2&-5\\-5&4\end{matrix}\right)$
4. $\left(\begin{matrix}2&0\\3&2\end{matrix}\right)$
5. The matrix in 4 (b) is a matrix of a particular type. All matrices of this type have real eigenvalue (for a proof, see Theorem 1 in http://www.quandt.com/papers/basicmatrixtheorems.pdf). Which of the two matrices below is of the same type, and what is that type called? Confirm your guess by finding the Eigenvalues of both matrices.
6. $\left(\begin{matrix}1&-2&0\\-2&2&5\\0&5&1\end{matrix}\right)$ b) $\left(\begin{matrix}1&2&0\\3&0&5\\1&-1&1\end{matrix}\right)$
7. Find all solutions to the homogeneous system of linear differential equations X’ = A X, where
8. A is the matrix in 4 (b)
9. A is the matrix in 5 (a)

For a nice, but perhaps a little lengthy, background info about the theory behind this HW, make sure to check: <http://www.math.psu.edu/tseng/class/Math251/Notes-LinearSystems.pdf>