**Recursive Function**s

Recursive function: Defined as a function calling itself!

F(x) := …. F(x-1) ….

Note: When the function calls itself, its argument must differ from the argument with which it was called. Usually it is something like n-1. In addition, there must be at least one branch of the function that does *not*include a recursive call.

Fact(n) :=

If[ (n == 1)

1   
else  
 n \* Fact(n-1)

With recursve functions you can program small miracles, in a few lines. Here is a Mathematica program that will solve the Tower of Hanoi problem. The idea is:

If (N >=1)

* Move n-1 disks from peg “from” to peg aux”
* Move nth disk from peg “from” to “to”
* Move n-1 disks from peg “aux” to peg “to”

The actual function looks like this:

hanoi[n\_, from\_, to\_, aux\_] :=  
If [ n >= 1,  
 hanoi[n-1, from, aux, to];  
 Print[“Move disk “, n, “ from “, from, “ to “, to];  
 Hanoi[n-1,aux, to, from] ];

This will solve the problem, but it is a double-recursion, which is O(2^n). Thus, to move 64 disks requires 2^64 = 1 steps, which would take billions of years indeed!!!!

**Homework**: try to guess big-O of our simplex algorithm!!!

Download our complete simplex algorithm from our web site. Run the simplex method 5 timed for n variables and n/2 random inequalities (there is some code that will do this). Pick n = 100 and find the average time it takes. Then do this for n = 300, n = 500, n= 700, n= 900, n=1100, etc. Create a chart with these runtimes and try to see what function might fit the data best.

**Differential Equations**

A DE (differential equation) or ODE (ordinary diff. eq.) is an equation that contains both an unknown function as well as the derivative of that function. A PDE is an equation that includes a function of two or more variables as well as some partial derivatives of that function. For example, all functions *u(x, y)* for which is a PDE. We will not deal with PDE’s but instead focus on ODE’s.

**Example:** dy/dt = k y(t) is a DE modelling exp growth (k > 0) or decay (k < 0). It model the situation where the growth of a function (the size of a population) is proportional to the function itself (the current size of the population). That makes sense: the more animals there are, the faster they grow in number. This is an example of a separable DE and can be solved easily:

Applying the exponential function on both sides gives

This is our familiar model for exp growth or decay. However, that is a simplistic model, because it implies that as time goes to infinity, the population increases without bound. In reality, though, scarcity of resources (like food) means that a population grows until it reaches a limit L, then it shrinks, giving the resource(s) a chance to regenerate, then it grows again, etc.

To model this more sophisticated situation we say: growth a population depends on its current size: if size is less than some number L, population grows, otherwise it shrinks. L is called the carrying capacity.

As a DE we might choose: y’ = k y (L – y). Thus:

dy/dt = k y (L-y) 🡺 🡺

**Homework 2**: Find (a) the function y(t) and (b) its limit y(t) as t 🡪 infinity. HINT: Use the fact that (or whatever other trick you happen to know.