**Real Analysis: Chapter 6**

1. Find a function defined for all and a sequence *{xn}* such that *xn* converges to *4* but *f(xn)* does *not* converge to *f(4)*
2. Use the epsilon-delta definition of the limit of a function to show that for .
3. True or false (if true, prove it, if false, give counterexample)
   1. If f is continuous, the *image* of an open set is again open
   2. If f is continuous, the *inverse* image of a connected set is connected
   3. If f is continuous, the *inverse* image of a compact set is compact
   4. A continuous function defined on a compact set is bounded
   5. A continuous function defined on a bounded set is bounded
   6. A continuous function defined on a closed set is bounded
   7. If is continuous, then must be continuous
4. Decide (with proof) where, if anywhere, the following functions are continuous (check the text):
   1. f(x) = 1 if x < 0 and f(x) = 0 otherwise
   2. g(x) = 1 if x is rational and g(x) = 0 if x is irrational
   3. h(x) = x if x is rational and h(x) = 0 if x is irrational
   4. k(x) = 1/q if x = p/q is rational and k(x) = 0 if x is irrational *Hint: Use a lemma (with proof) that if converges to any number c, then converges to infinity*
5. The function is continuous on . Is it uniformly continuous on that interval?
6. Consider the functions below. Are they continuous, or do they have a removable discontinuity, a jump discontinuity, or an essential discontinuity at the point where the function splits up:
   1. b)
   2. d)
7. Pick an arbitrary number and define for all . Show that f is uniformly continuous.
8. Show that the function is in but not in
9. Find a function that is twice differentiable but the second derivative is not continuous.
10. Define for and . Show that for for all , where is a polynomial. Use that fact to show that is a function and for all .
11. Prove that has exactly one solution in the interval