**Math 3515 – Homework 2**

1. True or false
	1. If $A⊂B$ and $card\left(A\right)=card(B)$ then $A=B$
	2. If $A×B$ is finite, then both *A* and *B* are finite
	3. If $A×B$ is infinite, then both *A* and *B* are infinite
2. Prove that
	1. $card\left( \left(-1,1\right)\right)=card( \left(0,1\right) )$
	2. $card\left( \left(a,b\right)\right)= card\left( \left(0,1\right)\right)$
	3. $card\left( R\right)=card\left( \left(0,1\right)\right)$
	4. $card\left( \left(0,1\right)\right)= card( \left(1,\infty \right))$
	5. $card\left( \left(0,1\right)\right)= card( \left[0, 1\right])$
3. What is the cardinality of the *countable* crossproduct $N×N×N×…$
4. Let P be the set of all polynomials with integer coefficients. Find $card(P)$.
Hint: Define $P\_{n}$ to be the set of all polynomials of degree n with integer coefficients. Then find $card\left(P\_{n}\right)$ and finally relate the $P\_{n}$ with the original set $P$
5. Recall that a number $x\_{0}$ is called algebraic if it is the root of a polynomial with integer coefficients. If a number is not algebraic, it is called transcendental. For example, all rational numbers are algebraic, so are $\sqrt{2}, \sqrt[3]{5}$, etc. while $π$ and $e$ (Euler’s number) are transcendental. Prove that the algebraic numbers are countable. What about the transcendentals?
6. Prove that $1^{3}+2^{3}+3^{3}+…+n^{3}=\frac{1}{4}n^{2}\left(n+1\right)^{2}$
7. At a party there are n people milling about. Everyone shakes hands with everyone else exactly once. How many handshakes happen? Make sure to prove your answer.
8. Show that there is no rational number $x$ with $x^{2}=3$

NOTE: In class we said that card(P(S)) > card(S) for any set S, but we did not prove it. Please read through Theorem 2.2.5: Cardinality of Power Sets in our online text. However, there is nothing to turn it, just read the prove. It is good for you ☺