**Math 3515 – Assignment 1**

1. In class we proved the first distributive laws for sets (IRA Prop 1.1.3). Prove the second distributive law $A∩\left(B∪C\right)=(A∩B)∪(A∩C)$ and visualize it by drawing appropriate Venn diagrams.
2. In class we proved the first De Morgan law (IRA Thm 1.1.4). Prove the second De Morgan law $comp\left(\bigcup\_{j}^{}A\_{j}\right)=\bigcap\_{j}^{}comp(A\_{j})$. Could you visualize it using Venn diagrams?
3. (a) True or false: if x is divisible by 3, then $x^{2}$ is divisible by 3. If false, give a counterexample; if true, prove it.

(b) True or false: if $x^{2}$ is divisible by 3, then $x$ is divisible by 3. If false, give a counterexample; if true, prove it.

1. (a) Is every number of the form $y=2^{n}-1$ a prime number? *Note: those that are are called Mersenne Primes (see* [*https://primes.utm.edu/mersenne/*](https://primes.utm.edu/mersenne/) *for more info).*
(b) Is every number of the form $y=1∙2∙3∙5∙7∙…p\_{n}+1$, $p\_{n}$ prime, a prime number?
(c) Assume that there is a largest prime number P. Would the number $y=1∙2∙3∙5∙7∙…P+1$ be prime? Use that fact to prove that there are infinitely many prime numbers.
2. For the function $f:R\rightarrow R, f\left(x\right)=x^{2}-1$, find:
	1. $imag(f)$
	2. $imag(\left[0,2\right])$ and $imag(\left[-1,3\right])$
	3. $f^{-1}\left(\left[0,1\right]\right)$ and $f^{-1}\left(\left[-1,1\right]\right)$ and $f^{-1}\left(\left[-3,-2\right]\right)$
3. In class we defined a relation $r$ on the set $N×N$ by calling $(a,b)$ and $(a^{'},b^{'})$ related if $a+b^{'}=a^{'}+b$. For the resulting equivalence classes we defined addition and multiplication as follows:
	1. $\left\{\left(a,b\right)\right\}+\left\{\left(c,d\right)\right\}=\left\{\left(a+c,b+d\right)\right\}$
	2. $\left\{\left(a,b\right)\right\}⋅\left\{\left(c,d\right)\right\}=\left\{\left(ad+bc,ac+bd\right)\right\}$
4. List all elements related to (1,5), to (8,2), and to (7,7)
5. Find $\left\{\left(1,5\right)\right\}+\left\{\left(8,2\right)\right\}$ and $\left\{\left(1,5\right)\right\}⋅\left\{\left(8,2\right)\right\}$ as well as $\left\{\left(1,5\right)\right\}⋅\left\{\left(7,7\right)\right\}$ and $\left\{\left(8,2\right)\right\}⋅\left\{\left(6,6\right)\right\}$
6. Find a better notation for the equivalence classes$ \left\{\left(1,5\right)\right\}, \left\{\left(8,2\right)\right\}$ and $\{(7,7)\}$
7. Prove that the relation $r$ on $N\_{0}×N$ defined via $\left(a,b\right)\~(a^{'},b^{'})$ if $ab^{'}=ba'$ is an equivalence relation. Here $N\_{0}=N∪\left\{0\right\}=\{0,1,2,3,…\}$
8. Show that if we change the domain of the relation in (7) to $N\_{0}×N\_{0}$ it is no longer an equivalence relation.
9. For the relation defined in (7), define $\left\{\left(a,b\right)\right\}⋅\left\{\left(c,d\right)\right\}=\left\{\left(ac,bd\right)\right\}$.
	1. Show that $\left\{\left(1,2\right)\right\}⋅\{\left(5,2\right)\}$ = $\left\{\left(3,6\right)\right\}⋅\{\left(10,4\right)\}$
	2. Show that the multiplication is indeed well-defined