**Math 3515 - Final Exam**

*This is a take-home exam. You may use your notes or the (online) textbook, but you must complete the*

*exam on your own. Please submit the completed exam either in person or via email on the scheduled day of our final.*

1. Give an example for each of the following. You do **not** have to prove your assertion.
	1. A function that is continuous everywhere except at x = 0 and x = 2 (a graph would suffice)
	2. A function that is not continuous anywhere
	3. A function that is continuous everywhere, and differentiable everywhere except at x = 0 and x = 2 (a graph would suffice)
	4. A function that is differentiable everywhere, but whose derivative is not continuous
	5. A function that is twice differentiable but not three times differentiable
2. Decide whether the following statements are true or false. If false give a counterexample, if true justify your decision.
	1. If the integral  = 0, then f(x) is identically zero on the interval [a, b]
	2. If a sequence of differentiable functions  converges uniformly to *f(x)*, then the limit function *f*  is continuous.
	3. If a sequence of differentiable functions  converges uniformly to *f(x)*, then the limit function *f* is differentiable.
	4. Let . If converges, then converges.
	5. There exists a monotone increasing function that is discontinuous at every irrational number.
	6. If f is differentiable on the interval [a, b], then f is uniformly continuous on that interval.

3. Consider the sequences of functions below. In each case, find the limit function and explain why the convergence cannot be uniform. (Hint: consider the implications of uniform continuity for the limit function, at least in case (a) and (b))

a) , where  for 

b) , where  for 

**Extra Credit:** , where  for 

4. Which of the following series is conditionally convergent, absolutely convergent, or divergent ? Justify your conclusions.

a)  b)  c)  ,  *(Hint: consider several cases; apply the ratio test)*

5. Each of the functions below has a discontinuity at x = 0. Determine whether it is removable, a jump, or essential.

a)  b) 

6. Let *f* be twice continuously differentiable. Prove that if *r*, *s*, and *t* are three roots of the equation *f(x) = 0* with *r < s < t*, then the equation *f’’(x) = 0* has at least one solution in the interval *(r, t)*.

7. Find the center and the radius of convergence of $\sum\_{n=1}^{\infty }\frac{1}{n}\left(2x-1\right)^{n}$. Also discuss convergence at the endpoints.

8. We know that $\sum\_{n=0}^{\infty } x^{n}=\frac{1}{1-x}$ for all $\left|x\right|<1$, and in class we deduced that $\sum\_{n=1}^{\infty }nx^{n}=\frac{x}{\left(1-x\right)^{2}}$ and $\sum\_{n=1}^{\infty }n^{2}x^{n}=\frac{x(1+x)}{\left(1-x\right)^{2}}$. Find a similar closed form for $\sum\_{n=1}^{\infty }n^{3}x^{n}$ for all $\left|x\right|<1$. (*Hint: To check your answer, you could use the fact that* $\sum\_{n=1}^{\infty }^{n^{3}}/\_{2^{n}}=26$)