**Real Analysis Highlights**

You need to be able to state or explain each of the following statements

**Chapter 1 to 3:**

* De Morgan Laws
* Euclid’s Theorem
* Equivalence Relations
* The Integers
* The Rationals
* Cardinality
* Countable and uncountable
* Cantor’s diagonalization argument
* Cardinality of **N**, **Z**, **Q**, **R**, **R**x**R**, algebraic numbers, transcendental numbers, irrational numbers
* Cardinality of countable unions of countable sets, finite cross products of countable sets, countable cross products of countable sets, power set of a set
* Continuum hypothesis
* Hilbert’s Hotel
* Cantor Bernstein Theorem
* Induction
* Partially ordered, ordered, well ordered
* Upper bound, lower bound, least upper bound, largest lower bound
* Least upper bound property of **R**
* Archimedian Property
* Sequence
* Convergence of a sequence
* Monotone or bounded sequence
* Cauchy sequence
* Lim sup and lim inf
* Monotone and bounded sequences converge
* Euler’s number
* Completeness theorem of **R**
* Subsequence
* Bolzano Weierstrass theorem
* Power sequence
* Exponent sequence
* Root n of n sequence
* Nth root sequence
* Binomial sequence
* Euler’s sequence
* Exponential sequence

**Chapter 4: Series**

* Series, including convergence of a series
* Absolute and conditional convergence
* Rearranging absolutely and conditionally convergent series
* *For fun:* Leaning Tower of Lire
* *For fun:* Zeno’s Paradox
* Convergence Tests
  + Divergence Test
  + Comparison Test
  + Limit Comparison Test
  + Root Test
  + Ratio Test
  + Alternating Series Test
  + Integral Test
* Special Series
  + Geometric Series (incl. proof)
  + Harmonic Series (incl. proof)
  + Alternating Harmonic Series
  + p Series
  + Euler’s Series

**Chapter 5: Topology**

* Open and Closed Sets
* Unions and Intersections of Open and Closed Sets
* Characterizing open sets
* Boundary Point and Interior Point
* Isolated Point and Accumulation Point
* Closed sets, accumulation points, and sequences
* Compact Sets
* Heine Borel Theorem
* Intersection of Nested Compact Sets
* Perfect Sets
* Perfect Sets are Unountable
* Connected and Disconnected sets
* The Cantor Set
  + is perfect
  + uncountable but zero length
  + does not contain any open set
  + is totally disconnected

**Chapter 6**

* Limit of a function (sequence version)
* Limit of a function (epsilon-delta version)
* Continuity
* Uniform Continuity
* Continuity vs Uniform Continuity
* Continuity preserves Limits
* Uniform Continuity preserves Cauchy sequences
* Types of Discontinuity
* Monotone functions and discontinuities
* Characterization of Discontinuity of the Second Kind
* Continuity and Topology (continuous functions and inverse images of sets)
* Images of compact sets and of connected sets
* Max/Min Theorem for continuous functions
* Bolzano Theorem
* Intermediate Value Theorem
* Derivative
* Derivative as linear approximation
* Differentiability and Continuity
* Product, Quotient, and Chain Rule
* Rolle’s Theorem and Mean Value Theorem
* L’Hospital Rules
* A function that is not continuous at any point in R (Dirichlet function)
* A function that is continuous at the irrational numbers and discontinuous at the rational numbers.
* A function that is differentiable, but the derivative is not continuous
* A function that is n-times differentiable, but not (n+1)-times differentiable (Cn function)
* A function that is not zero, infinitely often differentiable, but the n-th derivative at zero is always zero (Cinf function)
* A function that is continuous everywhere and nowhere differentiable in R (Weierstrass function)

**Chapter 7:**

* Partition and Riemann Sum
* Upper and Lower Sum
* Upper and Lower Riemann Integral
* Riemann Integral
* Riemann’s Lemma
* A function that is not Riemann integrable
* Continuous functions and the Integral
* Almost Continuous Functions and the Integral
* Monotone Functions and the Integral
* Properties of the Riemann Integral
* Fundamental Theorem Calculus
* Integral Evaluation Shortcut
* A function f such that is not differentiable
* Antiderivative
* Substitution Rule
* Integration by Parts
* Mean Value Theorem for Integration
* Partial Fraction Decomposition
* Be sure to know how to evaluate the following integrals:
  + and and
  + and