**Math 3515: Sample Midterm Exam**

*Please complete* ***any 6*** *of the* ***9*** *problems! More than 6 problems count as extra credit (specify your “basic 6”).*

1. Prove that if ***A*** is uncountable and ***B*** is countable, then ***A – B*** (***A*** minus ***B***) is uncountable.
2. Prove that the set ***D*** of **dyadic** **rationals**, deﬁned as , is countable. You ***may not*** use the fact that the rationals are countable.   
     
   *Note that the dyadic rationals are dense in* ***R****, i.e. between any two real numbers a and b there is a dyadic rational d such that a < d < b. This is just a nifty fact of life, nothing for you to do.*
3. ***Prove*** that
4. Find the ***inf***, ***sup***, ***lim inf***, and ***lim sup*** for the sequence .  *Note: if is the n-th prime number, then it is an unproven conjecture that . Just another neat fact ☺*
5. Define a **recursive** **sequence** as follows: and for *n > 1.* Show that the sequence must have a limit and find it.
6. **Prove** that if converges, then converges.Is the opposite also true, i.e. if converges then converges?
7. Are the following series **absolute convergent**, **conditionally convergent**, or **divergent** (justify your answer):
8. Find examples that illustrate each of the following questions:
   1. We know that any union of open sets is open. What about the **union of *closed* sets**, is that always closed?
   2. We know that if are compact and then the intersection is not empty. What about the intersection of nested **open** sets, can that be **empty**?
9. Prove that if ***C*** is a compact subset of **R**, then *sup(****C****)* is part of the set. In other words if ***C*** is compact, then *sup(****C****) = max(****C****)*.