**Real Analysis HW 10**

1. We know that the Dirichlet function is not Riemann integrable, while the function *f(x) = 1/q if x = p/q is rational and f(x) = 0 if x is not rational* is Riemann integrable and the Riemann integral is zero over the interval [0, 1]. What about

Is that function integrable? If so, what is the integral of f over [0, 1]

1. Prove that is Riemann integrable over the interval [0, 2] using the **definition** of Riemann integration. Then prove that is Riemann integrable over the interval [0, 2] using the **Riemann lemma**.
2. The function has only *one* point of discontinuity on the interval . It should therefore be Riemann integrable on that interval? But *is it* really integrable? If not, explain why not and why this does not violate the theorem that a function that is \*almost\* continuous is integrable.
3. The function is defined via . Graph *Si(x)* for *x > 0* by finding critical points, extreme values, inflection points, etc. (if any).
4. Prove that for any function *f* that is continuously differentiable.
5. Suppose *h(x)* and *k(x)* are continuous on [a, b] and . Prove that there exists a number such that . *Hint: Use the Mean Value Theorem 7.2.8 for Integration with and .*
6. Find a function such that is not Riemann integrable but is Riemann integrable.  
   *Hint: Look at a function that is somewhat similar to the Dirichlet function .*
7. Show that if is Riemann integrable then is also Riemann integrable.   
   *Hint: If then for all . Then estimate in terms of for a given partition.*
8. Find the following integrals:
   1. b.

c. d.

e.