# Real Analysis - Homework 08

(1) Consider the set $[-1,2) \cup \{\frac{2n+1}{n}\}$, i.e. the interval from -1 (included) to 2 (excluded), together with the point {3/1, 5/2, 7/3, 9/4, …}. Is it open or closed or neither? Which points are interior points and boundary points? Which points are isolated and which ones are accumulation points?

(2) Are the following sets open, closed, or neither:

a. Q
b. N
c. $\left\{1+\frac{1}{4}+\frac{1}{9}\ldots+\frac{1}{n^2} : n \in \boldsymbol{N}\right\}$

(3) True or false:

a. If $\{A_\alpha\}$ is a collection of closed sets, then $\bigcap_\alpha A_\alpha$ is closed
b. If $\{A_\alpha\}$ is a collection of closed sets, then $\bigcup_\alpha A_\alpha$ is closed
c. If *p* is an isolated point of *S* then *p* is a boundary point of *S*
d. If *x* is an interior point of *S*, then *x* is an accumulation point of *S*
e. If *x* is a boundary point of *S*, then *x* is an accumulation point of *S*
f. If *s = sup(S)*, then *s* is an accumulation point of *S*
g. If *L = lim* $(a_n)$, then *L* is an accumulation point of $\{a_n\}$
h. If *s = sup(S)* and *s* is not in *S*, then *s* is an accumulation point of *S*

(4) Which sets are compact?

a. $\boldsymbol{Q} \cap [0,1]$
b. $\boldsymbol{Z} \cap [0,10]$
c. $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\right\}$
d. $\left\{1,\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6},\ldots\right\}$

(5) Show that if *C* is compact and *F* is closed, then $C \cap F$ is compact

(6) Show that if *C* is compact then *sup(C)* and *inf(C)* both exist and are elements of *C*

(7) Find a collection of sets $A_n$ such that each $A_n$ is closed and not empty, $A_{n+1} \subset A_n$, but the intersection of all the $A_n$ is empty.

(8) Find a collection of sets $A_n$ such that each $A_n$ is open and not empty, $A_{n+1} \subset A_n$, but the intersection of all $A_n$ is closed and nonempty.