

Real Analysis - HW #4

Note Title

9/15/2012

- ① Show that $x^2 = 3$ has no solution in \mathbb{Q}
- ② Show that $x^2 = 3$ has a solution in \mathbb{R} .
- ③ Prove that $\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0$

Hint: use induction to show $\frac{n}{2^n} < \frac{1}{n} \forall n \geq 4$

- ④ We know that if $\{a_n\}$ converges, it is bounded. What about the converse?

- ④ If $a_n < b_n \forall n$ and $\lim_{n \rightarrow \infty} a_n = L_1$ and $\lim_{n \rightarrow \infty} b_n = L_2$, does this imply that $L_1 < L_2$?

- ⑤ Suppose $\lim_{n \rightarrow \infty} a_n = c$ and $\lim_{n \rightarrow \infty} b_n = d$. Then

a) prove that $\lim_{n \rightarrow \infty} (a_n + b_n) = c + d$

Hint: simple application of triangle inequality

b) prove that $\lim_{n \rightarrow \infty} (a_n \cdot b_n) = c \cdot d$

Hint: add zero in suitable form; also use

that $\{a_n\}$ and $\{b_n\}$ are bounded

- ⑥ Prove the "Squeezing Thm": if $a_n < b_n < c_n \forall n$ and $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = L$ then $\lim_{n \rightarrow \infty} b_n = L$