**Real Analysis Midterm Sample Questions**

1. The *closure(A)* is the set A together with its boundary points. The *interior(A)* is the set of all interior points of A. True or false: *interior(closure(A)) = A*.
2. If {xn} converges to x and {yn} converges to y, then prove:
	1. xn + yn converges to x + y
	2. xn \* yn converges to x \* y
3. True or false:
	1. If {xn + yn} and {xn} converges, then {yn} converges
	2. If {xn} converges then every subsequence converges
	3. If {xn} diverges then every subsequence diverges
4. Prove that if A and B are compact then $A∪B$ is compact. Is that true for countable unions, i.e. is it true that if An is compact for all n, then $\bigcup\_{n=1}^{\infty }A\_{n}$ is compact?
5. Show that any countable set in R has empty interior. How about the converse, i.e. if U is a subset of R with empty interior, then U must be countable?
6. Let $x\_{1}=1$ and $x\_{n+1}= \sqrt{1+2x\_{n}}$. Prove that {xn} converges and find the limit.
7. Prove that lim (2n + 1) / (n+4) = 2
8. Define Bert’s set by starting with B0 = [0, 1] and removing the middle 5th at each step. Then let $B=\bigcap\_{n=0}^{\infty }B\_{n}$. Show that B is uncountable but length of (B) is zero.
9. Can you have the following:
	1. A subset of R that is not bounded but sup(A) is finite
	2. An infinite sequence without accumulation point
10. Give an example of a sequence of numbers such that lim sup(xn), lim inf(xb), sup(xn), and inf(xn) are four distinct numbers.
11. True or false: If $\sum\_{}^{}|a\_{n}| $converges, then $\sum\_{}^{}\left|a\_{n}\right|^{2}$ converges. How about without the abs. values, i.e. is it true or false that if $\sum\_{}^{}a\_{n} $converges, then $\sum\_{}^{}a\_{n}^{2}$ converges.
12. Show that every Cauchy sequence is bounded.
13. True or false: if E is a non-empty subset of R and sup(E) is finite, then there is a sequence in E that converges to sup(E).
14. Show that if E is a non-empty and compact subset of R, then sup(E) is part of E.
15. Find an example of a set A that is not closed and every point is a limit point.
16. Show that an uncountable set must contain at least one of its limit points.
17. Show that if $\{x\_{n}\}$ converges to x, then $\{\frac{x\_{1}+x\_{2}+…+x\_{n}}{n}\}$ converges to x. Note that the sequence of algebraic means generated from the {xn} is called Cesaro Means.
18. Find a sequence $\{x\_{n}\}$ whose Cesaro means converge, but the original sequence does not.
19. Show that if $\left\{x\_{n}\right\}$, $x\_{n}>0$ for all n, converges to x, then the sequence of geometric means $\left\{(x\_{1}∙x\_{2}∙x\_{3}∙…∙x\_{n}\right)^{\frac{1}{n}}\}$ also converges to x. *Hint: Look at Cesaro Means and try the natural log.*
20. Test for convergence:
	1. $\sum\_{n=1}^{\infty }\sqrt{n^{2}+n}-n$
	2. $ \sum\_{n=1}^{\infty }\frac{1}{n^{2}+\sqrt{n^{2}+1}}$
	3. $\sum\_{n=2}^{\infty }ln⁡(\frac{n+1}{n})$
	4. $\sum\_{n=1}^{\infty }\frac{100^{n}}{n!} $