Panel 1

$$\int \frac{2x}{1-x^2}\,dx$$

$u = 1-x^2 \quad du = -2x\,dx$

$$-\int \frac{1}{u}\,du$$

$$-\ln(u) + c$$

$$-\ln(1-x^2) + c$$

$$\int \underbrace{x^2}_{f(x)} \underbrace{\sin(x)}_{g'(x)}\,dx$$

$f(x) = x^2 \qquad g'(x) = \sin(x)\,dx$

$f'(x) = 2x\,dx \qquad g(x) = -\cos(x)$

$$= 2\int \underline{x\cos(x)} - x^2\cos(x)$$

1

Panel 2

$$= 2\int \underline{x\cos(x)\,dx} - x^2\cos(x)$$

$$= 2\int \cos(x)\,x\,dx - x^2\cos(x)$$

$f(x) = x \qquad g'(x) = \cos(x)\,dx$

$f'(x) = dx \qquad g(x) = \sin(x)$

$$x^2(-\cos(x)) + 2x\sin(x) - 2\int \sin(x)\,dx$$

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x) + C$$

$$= \underline{2x\sin(x) - (x^2-2)\cos(x) + C}$$

2

Panel 3

$\int e^x \sin(x)\,dx$

$u = \sin(x) \quad dv = e^x\,dx$

$du = \cos(x) \quad v = e^x$

$e^x \sin(x) - \int e^x \cos(x)\,dx$

$u = \cos(x) \quad dv = e^x\,dx$

$du = -\sin(x)\,dx \quad v = e^x$

$= e^x \sin(x) - [e^x \cos(x) + \int e^x \sin(x)\,dx]$

$\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)\,dx$

$+ \qquad\qquad +$

$2\int e^x \sin(x)\,dx = e^x(\sin(x) - \cos(x))$

$= \frac{e^x}{2}(\sin(x) - \cos(x)) + C$

3

Panel 4

Last Time

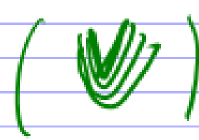
Remember Class

$f_n \to f$ pointwise

Problems:

f_n cont, $f_n \to f$: f not cont (x^n on $[0,1]$)

f_n diff'ble, $f_n \to f$: f not diff'ble



f_n int'ble, $f_n \to f$: f not int'ble $\quad f_n \to$ Dirichlet funct

f_n diff'ble, $f_n \to f$, f diff'ble: f_n' don't conv. $\frac{1}{n}\sin(nx)$

f_n int'ble, $f_n \to f$, f int'ble $\lim \int f_n\,dx \neq \int f\,dx$

$f_n = \max(n - n^2|x - \frac{1}{n}|, 0)$

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Panel 5

Uniform Convergence

Def. f_n converges uniformly to f iff for every $\varepsilon > 0$ $\exists N$ s.t. $|f_n(x) - f(x)| < \varepsilon$ $\forall n \geq N$ $\forall x \in D$

We write $f_n \rightrightarrows f$ on D

Ex. $f(x) = \frac{1}{n} x$, $x \in [0,2]$, $f_n \rightrightarrows 0$.

Proof. Take $\varepsilon > 0$, pick $N = 2/\varepsilon$. then

$$|f_n(x)| < \varepsilon \iff \frac{1}{n}|x| < \varepsilon \quad \forall x \in [0,2].$$

$$\forall n \geq N \Rightarrow n \geq 2/\varepsilon \Rightarrow \varepsilon \geq \frac{2}{n} \geq \frac{1}{n}|x| \quad \forall x \in [0,2]$$

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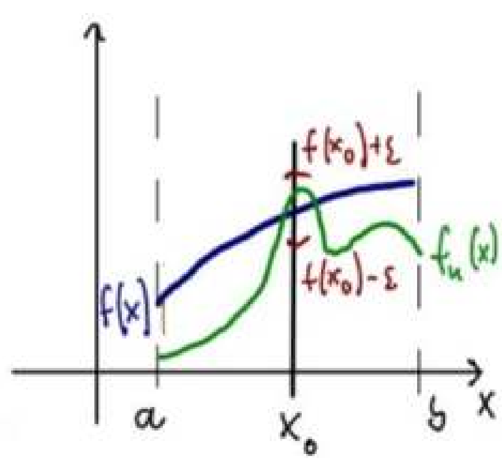
Panel 6

Pointwise Convergence

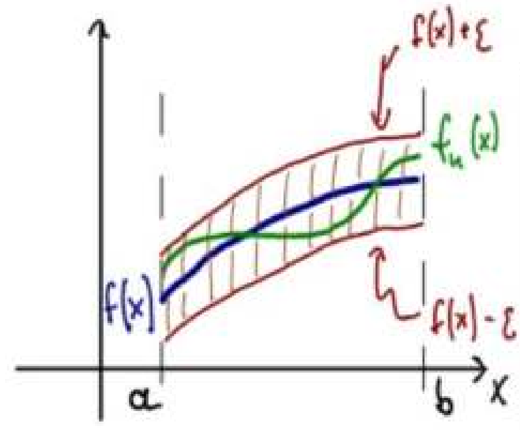
For pointwise convergence we first fix a value x_0. Then we choose an arbitrary neighborhood around $f(x_0)$, which corresponds to a vertical interval centered at $f(x_0)$.



Finally we pick N so that $f_n(x_0)$ intersects the vertical line $x = x_0$ inside the interval $(f(x_0) - \varepsilon, f(x_0) + \varepsilon)$

Uniform Convergence

For uniform convergence we draw an ε-neighborhood around the *entire* limit function f, which results in an "ε-strip" with $f(x)$ in the middle.



Now we pick N so that $f_n(x)$ is completely inside that strip *for all x in the domain.*

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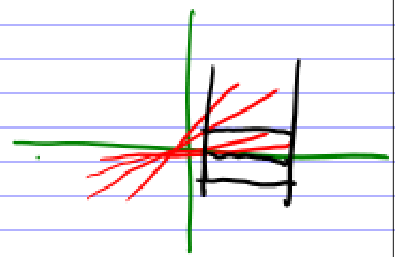
Panel 7

Ex: $f_n(x) = \frac{1}{n} x \quad , x \in [0,b]$

$f_n \to 0 \qquad f_n \rightrightarrows 0$



$f_n(x) = \frac{1}{n} x, \quad x \in \mathbb{R}$

$f_n \to 0 \qquad f_n \rightrightarrows 0$ NOT

Show this

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