Panel 1

Cont Thm

If f is intble $\Rightarrow$ $F(x) = \int_a^x f(t)\,dt$ is cont.

If f is cont $\Rightarrow$ $F' = f$

1

Panel 2

Corollary 7.1.19: Integral Evaluation Shortcut

Suppose *f* is an continuous function defined on the closed, bounded interval *[a, b]*, and G is a function on *[a, b]* such that *G'(x) = f(x)* for all *x* in *(a, b)*. Then

$$\int_a^b f(x)\,dx = G(b) - G(a)$$

The function *G* is often called *Antiderivative* of *f*, and this corollary is called *First Fundamental Theorem of Calculus*

Proof

Ex: $\int_0^1 x^3 - x^5\,dx = \frac{1}{4}x^4 - \frac{1}{6}x^6 \Big|_0^1 = \underline{\frac{1}{4} - \frac{1}{6}}$

2

Panel 3

Know: $F(x) = \int_a^x f(t)\,dt$ $\searrow$

$F(s) - F(a) = \int_a^s f(t)\,dt$ $F' = f$

Any $G(x)$ with $G'(x) = f(x)$.

$H(x) = F(x) - G(x) \Rightarrow H'(x) = F' - G' = f - f \equiv 0$

$\Rightarrow H(x) = F(x) - G(x) = c \Rightarrow G = F + c$

$G(s) - G(a) = \left(F(s) + c\right) - \left(F(a) + c\right) = F(s) - F(a) = \int_a^s f(t)\,dt$

$\#$

3

Panel 4

Example 7.2.2: Standard Antiderivatives

Find the following antiderivatives:

(a) $\int x^r\,dx$ $= \frac{1}{r+1}x^{r+1}$ (b) $\int 1/x\,dx$ $, \ln|x|$ (c) $\int e^x\,dx$ $, e^x$

(d) $\int \sin(x)\,dx$ $-\cos$ (e) $\int \cos(x)\,dx$ $\sin$ (f) $\int \tan(x)\,dx$ $-\ln(\cos(x))$

(g) $\int \frac{dx}{1+x^2}\,dx$ $\tan^{-1}(x)$ (h) $\int \frac{dx}{\cos^2(x)}\,dx$ $\tan(x)$ (i) $\int \frac{dx}{\sqrt{1-x^2}}\,dx$ $\sin^{-1}(x)$

$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$ $u = \cos(x)$, $du = -\sin(x)\,dx$

$= -\int \frac{1}{u}\,du = -\ln(u) + C$

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Panel 5

Substitution Rule: If f is continuous and s is cont. diff'ble ($s \in C^1$) function. Then

$$\int_a^b f(s(t))\, s'(t)\, dt = \int_{s(a)}^{s(b)} f(x)\, dx$$

Int. by Parts: If f, g are continuously diff'ble functions. Take $G(x) = f(x)\cdot g(x)$. Then

$$\int_a^b f(x)\, g'(x)\, dx = \left(G(b) - G(a)\right) - \int_a^b f'(x)\, g(x)\, dx$$

Proof: $G'(x) = \underbrace{f'(x) g(x) + f(x) g'(x)}_{\text{product}} \;\Big|\; \int \;\Rightarrow\; \int_a^b G'(x)\,dx = \int f'g + \int fg'$

$G(b) - G(a) =$

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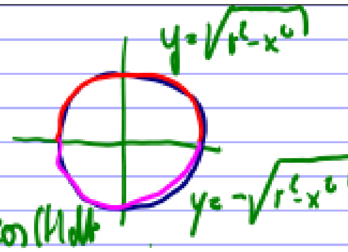
Panel 6

Ex. Find area of circle of radius r.



$$A = 2\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = 2\int_{-\pi/2}^{\pi/2} \sqrt{r^2 - r^2 \sin^2(t)}\; r\cos(t)\, dt$$

$$\boxed{\begin{aligned} x &= r\sin(t) \\ dx &= r\cos(t)\, dt \end{aligned}}$$

$$= 2\int r^2 \cos^2(t)\, dt = 2r^2 \int \cos^2(t)\, dt =$$

$$2r^2 \cdot \tfrac{1}{2}\pi = \underline{\pi r^2}$$

Recall: $\cos^2(x) = \frac{1}{2}(\cos(2t) + 1)$

$$\int \cos^2 dt = \frac{1}{2}\int \cos(2t) + 1\, dt = \frac{1}{4}\sin(2t) + \frac{1}{2}t \Big|_{-\pi/2}^{\pi/2} = 0 + \frac{1}{2}\pi$$

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Panel 7

Ex: $\int_a^b \underset{g}{x}\,\underset{f'}{e^x}\,dx = xe^x\Big|_a^b - \int_a^b e^x\,dx = xe^x - e^x\Big|_a^b$

Ex: $\int \underset{f'}{1}\cdot \underset{g}{\ln(x)}\,dx = x\ln(x) - \int x\,\frac{1}{x}\,dx = x\ln(x) - x + C$

Ex: $\int \underset{g}{x^2}\,\underset{f'}{\cos(x)}\,dx = x^2\sin(x) - \int 2x\,\underset{f'}{\sin(x)}\,dx$ flw

$= x^2\sin(x) - \left[-2x\cos(x) + \int 2\cos(x)\,dx\right]$

$= x^2\sin(x) + 2x\cos(x) - 2\int \cos(x)\,dx$

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Panel 8

$$\int \frac{2x}{(x^2+2)(x-3)}\,dx$$

$(x^2+2)(x-1)^2(x+5) \Rightarrow (x^2+2) + (x+5) + (x-1) + (x-1)^2$

$$\frac{2x}{(x^2+2)(x-3)} = \frac{Ax+B}{x^2+2} + \frac{C}{x-3} = \frac{(Ax+B)(x-3) + C(x^2+2)}{\#} = \frac{2x}{\#}$$

① x^2: $A + C = 0$

x: $B - 3A = 2$

const: $-3B + 2C = 0$

② pick $x=3$: $11C = 6 \Rightarrow C = \frac{6}{11}$

$x=0$: $-3B + 2C = 0 \Rightarrow B = \frac{2C}{3} = \frac{4}{11}$

$x=1$: $(A+B)(-2) + C(3) = 2 \Rightarrow A = -\frac{6}{11}$

$$\int \frac{2x}{(x^2+2)(x-3)}\,dx = \int \frac{-\frac{6}{11}x}{x^2+2}\,dx + \int \frac{\frac{4}{11}}{x^2+2}\,dx + \int \frac{\frac{6}{11}}{x-3}\,dx$$

$$= -\frac{6}{11}\int \frac{x}{x^2+2}\,dx + \frac{4}{11}\int \frac{1}{x^2+2}\,dx + \frac{6}{11}\int \frac{1}{x-3}\,dx =$$

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Panel 9

$$= -\frac{6}{11}\int \frac{x}{x^2+2}\,dx + \frac{4}{11}\int \frac{1}{x^2+2}\,dx + \frac{6}{11}\int \frac{1}{x-3}\,dx =$$

$$= -\frac{6}{11}\frac{1}{2}\ln(x^2+2) + \frac{6}{11}\ln|x-3| + \frac{4}{11}\frac{1}{2}\int \frac{1}{1+\left(\frac{x}{\sqrt{2}}\right)^2}\,dx$$

$$\frac{2}{11}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

Done with 7.1, 7.2 Rest of 7 in next semester

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Panel 10

Chapter 8: Function Sequences.

Sequences of functions:

how to define convergence?

is limit a function?

if each function is cont., limit cont.?

diff'ble diff'ble

int'ble int'ble?

Def: A function of 2 variables $f: D \times S \to \mathbb{R}$, $D \subset \mathbb{R}$, where second variable is called parameter, is a family of functions, $\{f_c(x)\}$.

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Panel 11

Ex: $\{f_c(x)\} = \{x^2 + c\}$ Domain: $x \in \mathbb{R}$, $c \in \mathbb{R}$

$\{f_n(x)\} = \{\frac{n}{x-n}\}$ Domain: $\mathbb{R} - \mathbb{N}$ $c = n \in \mathbb{N}$

First one has uncountable many members, 2nd countably many

A countable family of functions is called

Function Sequence

fix parameter: ⇒ get function

Ex: $\{f_n(x)\} = \{x^n\}$

fix x: ⇒ get a numeric sequence.

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Panel 12

Def: (Pointwise Convergence) $\{f_n(x)\}$ a seqn of functions.
It conv. pointwise if for each fixed x_0 the numeric sequence $\{f_n(x_0)\}$ converges.

Ex: $\{x^n\}$, $x \in [0,1]$. Find the limit function, if any.

if $0 \le x_0 < 1 \Rightarrow x_0^n \to 0$

if $x_0 = 1$

$$\Rightarrow f_n(x) = x^n \to g(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \end{cases}$$

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