Panel 1

Last Time

Heine-Borel Thm: cpct = closed + bdd (in $\mathbb{R}^n$)

Nested Compact Sets Thm: A_j compact, $A_{j+1} \subset A_j$
$\Rightarrow \bigcap_{j=0}^{\infty} A_j \neq \emptyset$

Perfect Sets: compact + every pt is accumulation [0,1]

Thm: Perfect sets are uncountable

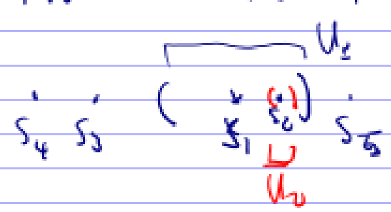
1

Panel 2

Every non-empty perfect set is uncountable.

Proof: Take $S \neq \emptyset$, S perfect, and assume that
$S = \{s_1, s_2, s_3, \ldots\}$ was countable

take $U_1 = (s_1 - 1, s_1 + 1)$ a nbhd. of s_1. Know s_1 is accum. point $\Rightarrow$ inf. many of the S are in U_1
Pick one of them, call it s_2. Find nbhd U_2 of s_2 s.t.

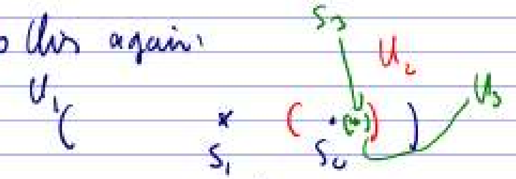


$\overline{U_2} \subset U_1$ — closure
$x_1 \in U_1$ but $x_1 \notin \overline{U_2}$

2

Panel 3

Do this again:



s_2 is accum. point $\Rightarrow$ pick $x_3 \in U_2$ and nbhd U_3 s.t.
$\overline{U}_3 \subset U_2$, $x_3 \in U_3$, $x_1, x_2 \notin \overline{U}_3$.

Continue: pick s_{n+1} and U_{n+1} s.t.
$s_{n+1} \in U_{n+1}$ $s_1, s_2, \dots, s_n \notin \overline{U}_{n+1}$, $U_{n+1} \subset U_n$

$V = \bigcap_{n=1}^{\infty} \left(\overline{U}_n \cap S\right)$. Then $\overline{U}_n \cap S$ are compact,
$\overline{U}_{n+1} \cap S \subset \overline{U}_n \cap S$, i.e. nested

$\Rightarrow V \neq \emptyset$ Take $t \in V$. $t \in U_2 \Rightarrow t \neq s_1$, $t \in U_3 \Rightarrow t \neq s_2$ ...
$\Rightarrow t \neq s_j$ $\forall j$. $S = \{s_1, s_2, \dots\}$ is not complete.

q.e.d.

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Panel 4

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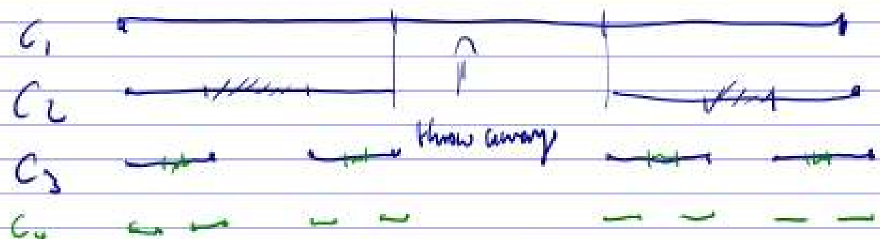
Panel 5

The Cantor Set

1.) Let $C_1 = [0,1]$ length of $C_1 = 1$

2) Let $C_2 = C_1 - (1/3, 2/3)$



Note: length of $C_2 = 1 - 1/3$

length of $C_3 = 1 - 1/3 - 2/9$

length of $C_4 = 1 - 1/3 - 2/9 - 4/27$

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Panel 6

Define $C = \bigcap_{n=1}^{\infty} C_n$ as the Cantor set

1. Is $C = \emptyset$
2. Is it finite or infinite
3. Are there # in C:
   a) $1/4$
   b) $8/27$
   c) $31/81$
4. Is C compact?

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