Panel 1

Topology

Worked with numbers, which had specific locations on number line. They were ordered, could count them etc.

Ex: $|a_n - c| < \varepsilon$

Now want to treat numbers as abstract entities forming ~~piles of stuff~~ sets

$\Rightarrow$ Topology

Def: A set $U \subset \mathbb{R}$ is open if for every $x \in U$ there is an ε-neighborhood $U_\varepsilon = (x-\varepsilon, x+\varepsilon) \subset U$.

A set is closed if its complement is open

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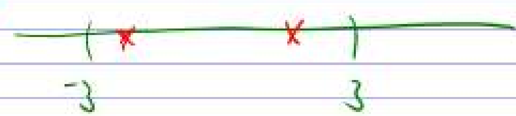
Panel 2

Ex: $(-3,3)$ - open: take $x_0 \in (-3,3)$, let $\varepsilon = \frac{3-x_0}{2}$, $(x_0-\varepsilon, x_0+\varepsilon)$

$[4,7]^c = (-\infty,4) \cup (7,\infty)$ $= \frac{x_0+3}{2}$ $\subset (-3,3)$

$(-4,5]$ neither



$(0,\infty)$ open

$[0,\infty)$ closed

$(-4,5]$ is not open, because 5 does not have ε-nbhd

is not closed because -4 doesn't have ε-nbhd in $(-4,5]^c$

$\mathbb{R}$: open $\mathbb{R}^c = \emptyset$ is open so $\mathbb{R}$ is open + closed

$\emptyset$: open and closed

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Panel 3

Thm: Every union of open sets is open

Every intersection of closed sets is closed

Every *finite* intersection of open sets is open

Every *finite* union of closed sets is closed

Q1: Find intersection of open sets that is *not* open

$\cap (0, 1+\frac{1}{n}) = (0,1]$

Q2: Find a union of closed sets that is *not* closed.

HW

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Panel 4

~~Thm~~: "Open sets are boring, closed sets can be interesting"

Ex: $\{1,2\}$ closed, $\{1,2\}^c = (-\infty,1) \cup (1,2) \cup (2,\infty)$

$\{1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots\}^c = (1,\infty) \cup (\frac{1}{2},1) \cup (\frac{1}{3},\frac{1}{2}) \cup \ldots (-\infty, 0]$

$\{1,\frac{1}{2},\frac{1}{3},\ldots\}^c$ includes 0, but *no* $(-\varepsilon,\varepsilon)$.

so $\{1,\frac{1}{2},\frac{1}{3},\ldots\}$ is not closed

$\{1,\frac{1}{2},\frac{1}{3},\ldots\} \cup \{0\}$ *is* closed

Thm: Every open set in $\mathbb{R}$ is a countable union of disjoint, open intervals!

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Panel 5

Def: S any set in $\mathbb{R}$.

1.) A point $x \in \mathbb{R}$ is a boundary point of S if every nbhd. of x intersects S and S^c

2.) A point $x \in S$ is an interior point of S if $\exists$ ε-nbhd contained in S

3.) A point $x \in S$ is called isolated if $\exists$ ε-nbhd. of x whose intersection with S is empty

4.) A point $x \in S$ is called accumulation point, if every nbhd of x contains infinitely many points of S

Ex. $(0,4)$

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