Panel 1

About Cantor's Diagonalization Argument

$S = \{$ infinite sequences of 0 and 1 $\}$

$s = (1,0,0,1,0,0,0,1,1,\dots) \in S$

S is uncountable! (like $(0,1)$ or $\mathbb{R}$)

$\hat{S} = \{$ finite sequences of 0 and 1 $\}$

$s \in (1,0,0,1,0,1,\underbrace{0,0,0,\dots}_{\text{zeros}})$

$\hat{S}$ is countable (like $\mathbb{Q}$)

Please read: http://en.wikipedia.org/wiki/Georg_Cantor

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Panel 2

Why is $\hat{S} = \{$ finite sequences of $0,1\}$ countable?

$S_n = \{ s \in \hat{S} : s_i = 0 \text{ for } i > n \}$

$S_1 = \{(0,0,0,\dots), (1,0,0,\dots)\}$

$S_2 = \{(0,1,0,0,0), (1,1,0,0,\dots)\}$

$\text{card}(S_n) = 2^n$

$(\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{n\text{-slots}}, 0,0,0,\dots)$

$\bigcup_{n=1}^{\infty} S_n = \hat{S}$ countable!

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Panel 3

Why is $S = \{$infinite sequ. of 0 and 1$\}$ uncountable

Suppose not. $\Rightarrow$ List all elements in S in order

$s_1 = (s_1^1, s_2^1, s_3^1, \ldots)$ e.g. $(0,1,1,0,0,1\ldots)$

$s_2 = (s_1^2, s_2^2, s_3^2, \ldots)$ $(1,0,1,1,1,1,0,1,0\ldots)$

$s_3 = (s_1^3, s_2^3, s_3^3, \ldots)$ $(0,0,0,0,1,1,0\ldots)$

$\vdots$

$\Rightarrow t = (1,0,1,\ldots)$

Define $t = (t_1, t_2, t_3, \ldots)$ s.t.

$$t_i = \begin{cases} 0 & \text{if } s_{ii} = 1 \\ 1 & \text{if } s_{ii} = 0 \end{cases}$$

t is not equal to s_i, because they differ in i-th position

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Panel 4

Why can't you use Cantor's diag. argument to show that $\hat{S}$ is uncountable ???

List all elements from $\hat{S}$ in order

$s_1 = (s_1^1, s_1^2, \ldots)$ ex: $s = (0,1,0,0,1,0,0,0)$

$s_2 = (s_1^2, s_2^2, \ldots)$ $s_2 = (1,0,1,1,1,0,0,0)$

$s_3 = (s_1^3, s_2^3, \ldots)$

As before $t = (t_1, t_2, \ldots)$ with $t_i = \begin{cases} 0 & \text{if } s_{ii} = 1 \\ 1 & \text{if } s_{ii} = 0 \end{cases}$

Again, t is not any of the s_i's, so... contradiction ???

But t may not be finite

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