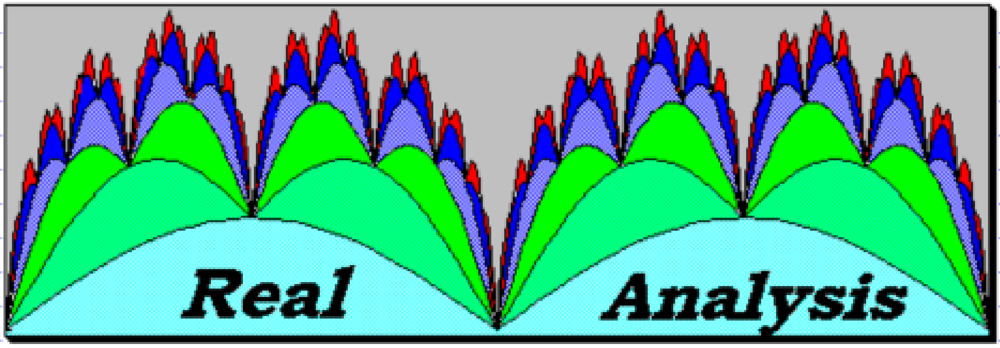
Panel 1



Welcome to Math 3515

with

Bert Wachsmuth

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Panel 2

The Essentials:

→ wachsmut@slu.edu

x5167

http://pirate.slu.edu/~wachsmut/

HW Install Dyknow ← Comm. Settings

vision.dyknow.com/slu.edu

username: 8-letter

pwd: same

Grade: 2 Exams 200

Homework 100

Presentation of HW

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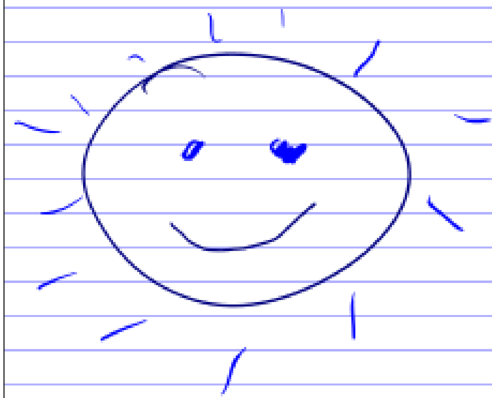
Panel 3

Dykuaw:



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Panel 4

Real Analysis Content

Review of Calc

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Panel 5

Set Theory

Notation: Set is a collection of objects from some universe.

$A \subset B$: subset: everything in A is also in B

$A \cup B$: union: in A, B, or both

$A \cap B$: intersection: in both A and B

$A \setminus B$: A minus B $(A-B)$ all from A not B

$\text{comp}(A) = A^c = \overline{A}$: all but A

disjoint: $A \cap B = \emptyset$

equal: $A \subset B$ and $B \subset A$

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Panel 6

$\mathbb{N} = \{1, 2, 3, \ldots\}$ natural start

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ integers

$\mathbb{Q} = \{p/q, \ p \in \mathbb{Z}, q \in \mathbb{N}\}$

$\mathbb{I}$ = irrationals

$\mathbb{R}$ = reals

$\mathbb{C}$ = complex #'s

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Panel 7

$A = \{x \in \mathbb{R} : -4 < x < 3\}$

$B = \{x \in \mathbb{R} : -1 < x < 7\}$

$I = \{x \in \mathbb{R} : x^2 = -2\}$

$\Rightarrow A \setminus B = \{x \in \mathbb{R} : -4 < x \leq -1\}$

$\text{comp}(A) = (-\infty, -4] \cup [3, \infty)$

$\text{comp}(I) = \text{comp}(\underbrace{\emptyset}_{\text{empty set}}) = \mathbb{R}$

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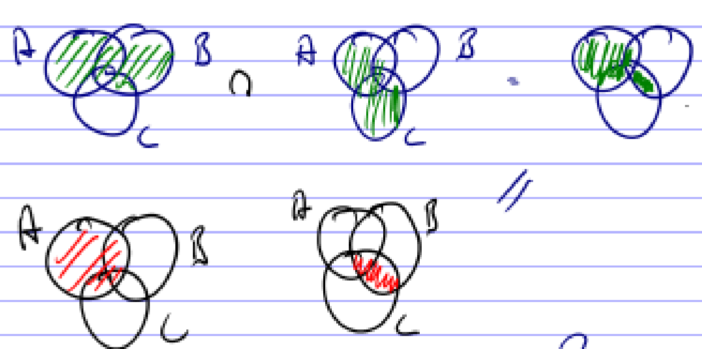
Panel 8

Prop: (Distributive Law)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:



colored lines - not a proof!

Real proof as HW

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Panel 9

<u>Thm</u> (De Morgan Laws)

(1) $\text{comp}\left(\bigcap_{j=1}^{\infty} A_j\right) = \bigcup_{j=1}^{\infty} \text{comp}(A_j)$

$\text{comp}\left(\bigcup_{j=1}^{\infty} A_j\right) = \bigcap_{j} \text{comp}(A_j)$

<u>Proof:</u> (1a) Take any $x \in \text{comp}\left(\bigcap_j A_j\right)$

$\Rightarrow$ x is <u>not</u> in intersection of A_j's

$\exists$ at least of A_k s.t. $x \notin A_k$

$\Rightarrow x \in \text{comp}(A_k)$

$\Rightarrow x \in \bigcup_j \text{comp}(A_j) \Rightarrow$ "$\subset$" ✓

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Panel 10

(1b) $x \in \bigcup_j \text{comp}(A_j)$

$\Rightarrow$ x is in at least one $\text{comp}(A_k)$

$\Rightarrow$ x is not in A_k

x is not in $\bigcap A_j$

$\Rightarrow x \in \text{comp}\left(\bigcap_j A_j\right)$ "$\supset$" ✓

<u>Done</u>

q.e.d.

quod erat demonstrandum

"what was to be proven"

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Panel 11

Other proving tech: induction, direct, constructive, by contradiction

<u>Thm:</u> Mult. of two odd int. gives odd int.

Say x is odd, i.e. $x = 2n+1$, $n \in \mathbb{N}_0$

y is odd, i.e. $y = 2m+1$, $m \in \mathbb{N}_0$

$xy = (2n+1)(2m+1) = 4nm + 2n + 2m + 1$

$= 2(2nm + n + m) + 1 = 2k+1$ odd.

q.e.d.

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Panel 12

<u>Thm:</u> If x^2 is even, then x is even.

<u>Proof:</u> Say $x^2 = 2n$, $n \in \mathbb{N}$ is even,

but x is not!

$\Rightarrow x$ is odd $\Rightarrow x = 2n+1$

$\Rightarrow x^2 = (2n+1)^2 = 4n^2 + 4n + 1 =$

$= 2(2n^2 + 2n) + 1$

$= 2k+1$ is odd

↯ contradiction $\Rightarrow$ q.e.d

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Panel 13

<u>Euclid's Thm:</u> There is no largest prime!

Say there is, i.e. M is the largest prime.

$a, b, c, d, \ldots$ are all primes $\leq M$

Take $x = a \cdot b \cdot c \cdot d \cdot \ldots \cdot M + 1$

x divisible by a: $\frac{x}{a} = bcd \ldots M + \frac{1}{a}$ no!

x not divisible by any prime! $\Rightarrow$ x is prime, $x > M$

$\Rightarrow$ q.e.d.

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Panel 14

Is $x = abc \cdots N + 1$ prime, if $a, b, c, \ldots$ are prime?

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