

Calc 3 - Assignment #3

Note Title

9/9/2011

① Which expression makes sense: (where \cdot is dot-product)

a) $\vec{a} \cdot \vec{b} \cdot \vec{c}$

b) $\|\vec{a}\|(\vec{b} + \vec{c})$

c) $\vec{a} + \vec{b} + \vec{c}$

d) $\vec{a} \cdot \vec{b} + \vec{c}$

e) $\|\vec{a}\|(\|\vec{b} \cdot \vec{c}\|)$

Note: $\text{comp}_{\vec{a}}(\vec{b}) = \|\text{proj}_{\vec{a}}(\vec{b})\|$

② Find $\text{proj}_{\vec{a}}(\vec{b})$ and $\text{comp}_{\vec{a}}(\vec{b})$ for

a) $\vec{a} = \langle 1, 2, -4 \rangle$ and $\vec{b} = \langle 5, 0 \rangle$

b) $\vec{a} = \langle 3, 6, -2 \rangle$ and $\vec{b} = \langle 1, 2, 3 \rangle$

③ The vector $\text{orth}_{\vec{a}}(\vec{b}) = \vec{b} - \text{proj}_{\vec{a}}(\vec{b})$ is called the orthogonal projection of \vec{b} . Show that

$\text{orth}_{\vec{a}}(\vec{b})$ is orthogonal to \vec{a} .

and draw a picture showing these vectors

④ Find the work done by a force $\vec{F} = 8\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}$

that moves an object from the point $(0, 0, 8)$ to the point $(6, 12, 20)$ along a straight line.

⑤ A sled is pulled along a level path through snow by a rope. A 30 lb force acting at an angle of 40° moves sled 80 feet. Find the work done by the force.

⑥ Find the angle between a diagonal of a cube and one of its edges.

7) Find the cross product $\vec{a} \times \vec{b}$ for

a) $\vec{a} = \langle 6, 0, -2 \rangle$, $\vec{b} = \langle 0, 9, 0 \rangle$

b) $\vec{a} = \langle 1, 1, -1 \rangle$, $\vec{b} = \langle 2, 4, 6 \rangle$

c) $\vec{a} = \langle t, t^2, t^3 \rangle$, $\vec{b} = \langle 1, 2t, 3t^2 \rangle$

8) Prove that $\vec{a} \times \vec{a} = 0$ and that

$\vec{a} \times \vec{b}$ is perpendicular to \vec{a} and to \vec{b}

9) Which expression does not make sense?

a) $\vec{a} \cdot (\vec{b} \times \vec{c})$

b) $\vec{a} \times (\vec{b} \cdot \vec{c})$

c) $\vec{a} \times (\vec{b} \times \vec{c})$

d) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

e) $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$

f) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

10) Is the cross product commutative, i.e. is

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{a} \quad (\text{try some examples first})$$

11) Find two unit vectors perpendicular to both

$$\langle 1, -1, 1 \rangle \text{ and } \langle 0, 4, 4 \rangle$$

12) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ does it follow that $\vec{b} = \vec{c}$?

If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does it follow that $\vec{b} = \vec{c}$?