**Fun with Mathematica**



We are currently covering functions whose graph is a surface in over the domain in the -plane. In the abstract such a graph looks like as shown on the right, but it is often difficult to visualize such a surface exactly. Thus, it pays to learn about the Mathematica functions Plot3D, as it can generate pretty cool looking surfaces.



**Example**: Draw the graph of



|  |  |
| --- | --- |
| If domain is assumed to be the square : |  |
| If we want to draw the surface for z between -2 and 24, say, use the PlotRange parameter |  |
| If the domain is assumed to be the disk of radius 3 rather than a rectangle [-3,3] x [-3,3], for example, use the RegionFunction parameter  as well as the BoxRatios:  or |  |
| If we want to draw two or more surfaces in one coordinate system, use curly brackets and separate the functions by commas. |  |

**Partial Derivatives and Gradient**

To compute a partial derivative, use the operator, as in for , for , etc. To compute the gradient of , use . To compute the partial derivative at a given point , add the standard replacement operator /. as in . Similarly, to find the gradient at a point, use .

For example: gives

1. **High and Low**

Plot the graph of the function over the square [-3, 3] x [-3, 3]. You will see that, the top and bottoms are cut off. Use an appropriate PlotRange parameter to show the entire graph.

*Hint: Be careful when you define the exponential function. Something like e^(-x^2-y^2) will* ***not*** *work (Mathematica will interpret the letter e as a variable, not as Euler’s constant e = 2.71…. Perhaps replacing e^x by the exponential function Exp will work]*

1. **Mexican Sombrero**

The “Mexican Sombrero” is the graph of , drawn over a square [-10,10] x [-10,10] or over the disk centered at the origin with radius 10, with z between -0.5 and 1. It looks like a Mexican sombrero (hence the name). Draw this function.

***Hint****: Aside from drawing the function for {x, -10, 10} and {y, -10, 10} you will need to specify the z-range by adding the parameter PlotRange -> {-0.5, 1}. If you want to see the hat over the disk with radius 10, you need to use the appropriate RegionFunction and perhaps set the BoxRatios -> Automatic.*

1. **The Goblet of Fire**

The function is a regular paraboloid with vertex at , while is a paraboloid going down, with vertex again at (0,0). If we draw the graphs of *both* functions in one coordinate system over the disk of radius 2, we get a figure that resembles a goblet (of fire). Draw this goblet.

*Note: The goblet shape will only become clear if you restrict the domain to a disk of radius 4 via the proper RegionFunction and BoxRatios->[1,1,1] or Automatic parameters. Try multiplying either the top or bottom function by 2 or 3 to vary the shape of your goblet. You of course need to draw both surfaces in the same plot.*

1. **Tangent Plane**

Draw the graph of the function together with its tangent plane at the point .

*Note: This should be pretty straight-forward (I hope).*

1. **Partial Derivatives**

Compute the x and y partial derivatives for

1. **Gradient**

Draw the contour plot for the function . Then compute the gradient of at and at (-2.5, -2.5) Draw the resulting vectors onto the contour plot to verify that they are perpendicular to the level curves.