**Fundamental Theorem of Line Integration: Consequences**

If is a *conservative* vector field with potential function , and is a curve from to , then:

Thus, if a vector field is *conservative*, we have *two* ways to find the work integral :

1. you can use the *definition* of the line integral (as long as the path is explicitly given), or
2. you could *find the potential function* and then compute the difference

The fundamental theorem of line integration has two important consequences:

**Corollary (Conservative Vector Fields over closed Loops)**

If is a conservative vector field and a closed curve, then:

In other words, the work done (i.e. the work integral) along a closed loop is zero. The symbol means of course that your path of integration is closed.

**Note:** The converse is true as well, in most cases: if the work for all closed loops, then is conservative.

Incidentally, this explains the term “conservative”: in a conservative vector field work is conserved: if you move in a circle, you expand as much work as you get back, leaving the total work to be zero.

**Corollary (Path Independence)**

If is a conservative vector field and , two curves, both going from points P to Q, then:

In other words, the work done (i.e. the work integral) does not depend on the particular path going from to , it just depends on here you start and where you stop.

**Example**: Which of the following vector fields is not conservative?

 

F = <-y,x> F = <-x, y> F=<x^2 y, y\*x^3>

**Example:** This, especially the first corollary, makes some work integrals especially easy to evaluate. Find , where , the circle around the origin with radius 2