**Vector Fields**

* 2D vector fields where
* 3D vector fields where

**Examples:** or

1. **Visualize**

Vector fields are everywhere: water flowing down a stream, magnetic field of the earth, electric fields, gravity, etc. As usual, the best and quickest way to draw a vector field is, of course, Mathematica.

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1. **Limits and Continuity**

A vector field is continuous iff and are continuous. So, nothing new here.

1. **Derivatives**

A 2D vector field has 4 partial derivatives: . How many partials does a 3D vector field have?

**Definitions**

* **Divergence:** With we define the *divergence* of a vector field as:
* **Curl:** With we define the *curl* of a vector field as:
* **Conservative:** A vector field is *conservative* if there is a function such that . The function is called the **potential** function of .

Note that the gradient of a function is a vector field, the divergence of a vector field , , is a function, and the curl of a vector field , , is a vector field.

What is the one-variable equivalent of a potential function?

**Problems to Work Out**

1. Let , find the divergence and the curl of

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1. If is a function and is a vector field, which of the following expressions make sense:
2. curl(f)
3. grad(f)
4. div(F)
5. grad(F)
6. curl(grad(f))
7. grad(curl(F))
8. grad(div(F))
9. div(grad(F))
10. grad(grad(f))
11. curl(curl(F))
12. div(div(F))
13. Prove the “product” rule:
14. Find a vector field that has the potential function . Is F conservative?
15. Prove that if a 3D vector field is conservative, then . What about a 2D conservative vector field?
16. Which of the following vector fields are not conservative?
17.
18.
19.
20. Find the potential function for if there is one
21. Find the potential function for if there is one
22. Find the potential function for if there is one