**Applications of Double Integration**

1. **Center of Mass**

Suppose $ρ(x)$ is the density of a thin lamina of shape D. Then:

* Mass: $m=∬\_{D}^{}ρ(x,y)dA$
* Moment about x-axis: $M\_{x}=∬\_{D}^{}y ρ(x,y)dA$
* Moment about y-axis: $M\_{y}=∬\_{D}^{}x ρ(x,y)dA$
* Center of gravity: $(\overbar{x},\overbar{y}) =(\frac{M\_{y}}{m},\frac{M\_{x}}{m})$

**Examples**: Find the center of mass of the following laminas (first guess, then work it out)

1. Square with corners (0,0) and (2,2) and uniform density $ρ\left(x,y\right)=k$
2. Square with corners (0,0) and (2,2) and uniform density $ρ\left(x,y\right)=x$
3. Triangle (0,0), (0,1), (2, 0) with density function $ρ\left(x,y\right)=1+3x+y$
4. A semi-circle of radius R whose density is proportional to the distance from the origin
5. **Moments of Inertia**

The moments of inertia, or second moments, tell us how much an object resists rotation around an axis. The two easiest ones are:

* $I\_{x}=∬\_{D}^{}y^{2}ρ(x,y)dA$ (2nd moment about x-axis)
* $I\_{y}=∬\_{D}^{}x^{2}ρ(x,y)dA$ (2nd moment about y-axis)

**Examples**: Which lamina has a larger moment of inertia about the y-axis: lamina is a rectangle with corners (0,0) and (2,4), density function is:

1. $ρ\left(x,y\right)=\frac{3}{32}x^{2}$
2. $ρ\left(x,y\right)= \frac{3}{64}(4-x^{2})$

Verify that both laminas have the same mass. Interpret your answer. How about inertia about the x-axis?

Give an example of a physical object with a large rotational moment of inertia