Summary 18: Absolute Extrema

Theorem: A continuous function on a closed, bounded set in R^n has an absolute max and an absolute min. Moreover, it can occur only at a critical point or on a boundary point.

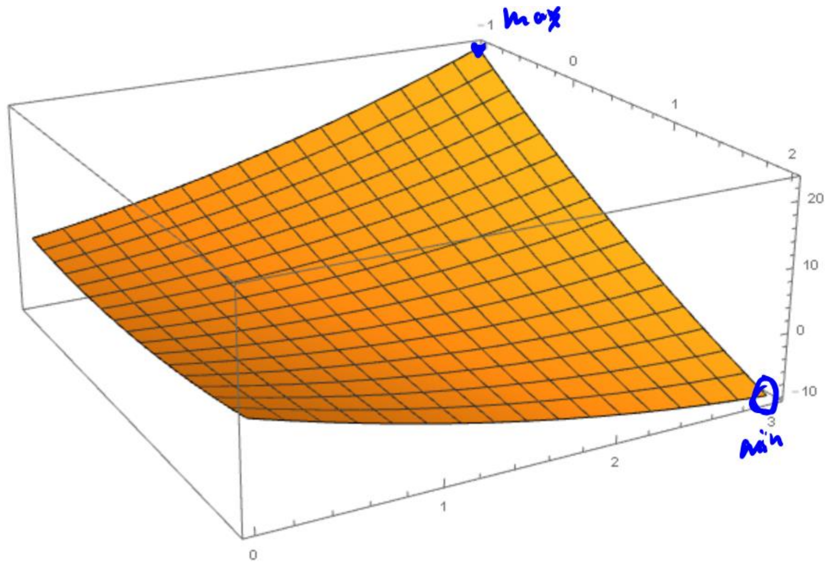
How to find absolute extrema

If f is a continuous function defined on closed and bounded set D:

1. Find the gradient ∇f
2. Solve system of equations $\nabla f = < 0,0 >$ to find critical points
3. Find all critical points along the boundary of D
4. Create a table evaluating f at all points found in (2) and (3) as well as any endpoints.

Then

a) The largest overall value is the absolute max
b) The smallest overall value is the absolute min



Find absolute extrema for

$$f(x, y) = x^2 + y^2 - 4xy + 1$$

on the set [-1, 2] x [0,3], i.e. for $-1 \leq x \leq 2$ *and* $0 \leq y \leq 3$

$f_x = 2x - 4y = 0$ so that $x = 2y$

$f_y = 2y - 4x = 0$ so that $2y - 8y = 0$

Critical point inside D: $(0,0)$

Checking the boundary:

$x = -1$: $f(-1, y) = y^2 + 4y + 2$

$f'(y) = 2y + 4 = 0$

Critical point $y = -2$

$x = 2$: $f(2, y) = y^2 - 8y + 5$

$f'(y) = 2y - 8 = 0$

Critical point $y = 4$

$y = 0$: $f(x, 0) = x^2 + 1$

$f'(y) = 2x = 0$

Critical point $x = 0$

$y = 3$: $f(x, 3) = x^2 - 12x + 1$

$f'(y) = 2x - 12 = 0$

Critical point x= 6

(x,y)	f(x,y)
(0, 0)	1
(-1, -2)	outside domain
(2,4)	outside domain
(3,6)	outside domain
(-1,0)	2
(2, 0)	5
(-1, 3)	23
(2, 3)	-10

Abs. max is 23, at (-1, 3)

Abs. min is-10 at (2, 3)