Summary 16: Partial Derivatives

Directional Derivative $D_u(f)$ of a function $f(x,y)$ in the direction of the unit vector $u = < a, b >$ is defined as $$D_u(f) = \lim_{h\to 0} \frac{f(x+ha, y+bh) - f(x,y)}{h}$$	
How to find $D_u(f)$: We have $D_u(f) =< f_x, f_y >< a, b >= af_x + bf_y$	*Find directional derivative of $f(x,y) = \frac{x}{y^2-1}$ in the direction $u =< \frac{1}{2}, \frac{\sqrt{3}}{2} >$ at the point (1,0):* $$\nabla f =< \frac{1}{y^2-1}, -\frac{2xy}{(y^2-1)^2} >$$ so that $\nabla f(1,0) =< -1,0 >$. Then $D_u(f) =< -1,0 > \cdot < \frac{1}{2}, \frac{\sqrt{3}}{2} >= -\frac{1}{2}$
Gradient of f: The gradient of a function $f(x,y,z)$ is the vector of all partials $grad(f) = \nabla f =< f_x, f_y, f_z >$. It has a number of properties: 1. ∇f applies to a function and results in a vector 2. ∇f is perpendicular to the level curves of f 3. ∇f points in the direction of largest increase, and $\|\|\nabla f\|\|$ is the largest increase 4. $D_u(f) = \nabla f \cdot u$, u a unit vector	*Suppose $f(x,y) = 4x^2 - y^3$ describes a small hill. You are standing at P(2,1). In which direction should you walk to go up the most, and what is the max increase:* $\nabla f =< 8x, -3y^2 >$. Thus $\nabla f(2,1) =< 16, -3 >$ and $\|\|\nabla f(2,1)\|\| = \|\| < 16, -3 > \|\| = \sqrt{265}$. So, steepest increase is $\sqrt{265}$ in the direction of $< 16, -3 >$
Partial Differential Equation: A PDE is an equation involving a function and some or all of its partials.	*Show that $f(x,y) = sin(y)\, e^x$ satisfies the Laplacian PDE* $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u_{xx} + u_{yy} = 0$$ $f_x = \sin(y)\, e^x$ and $f_{xx} = \sin(y)\, e^x$ $f_y = \cos(y)\, e^x$ and $f_{yy} = -\sin(y)\, e^x$ Thus: $f_{xx} + f_{yy} = 0$