Summary 14: Partial Derivatives

Partials: For $f(x,y)$ define partial derivatives:

$$\frac{\partial f}{\partial x} = f_x = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$\frac{\partial f}{\partial y} = f_y = \lim \frac{f(x,y+h) - f(x,y)}{h}$$

Use definition of partial derivative to find f_x and f_y for $f(x,y) = x^2y - 3xy$

$$f_x = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h} =$$
$$= \lim_{h \to 0} \frac{((x+h)^2y - 3(x+h)y) - (x^2y - 3xy)}{h} =$$
$$= \lim_{h \to 0} \frac{x^2y + 2xhy + h^2y - 3xy - 3hy - x^2y + 3xy}{h} =$$
$$= \lim_{h \to 0} \frac{2xyh + h^2y - 3hy}{h} =$$
$$= 2xy - 3y$$

$$f_y = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h} =$$
$$= \lim_{h \to 0} \frac{(x^2(y+h) - 3x(y+h)) - (x^2y - 3xy)}{h} =$$
$$= \lim_{h \to 0} \frac{x^2y + x^2h - 3xy - 3hx - x^2y + 3xy}{h} =$$
$$= \lim_{h \to 0} \frac{x^2h - 3hx}{h} =$$
$$= x^2 - 3x$$