Summary 11: Motion in R^3

Motion or path of a particle:

$$r(t) =< x(t), y(t), z(t) >$$

Velocity:

$$v(t) = r'(t) =< x'(t), y'(t), z'(t) >$$

Speed:

$$s = ||v(t)||$$

Acceleration:

$$a(t) = v'(t) = r''(t) =< x''(t), y''(t), z''(t) >$$

Also:

$$a = a_t T + a_n N$$

where:

$a_t = \frac{v \cdot a}{s}$ (tangential component)

$a_n = \frac{||v \times a||}{s}$ (normal component)

$\vec{T}(t) = \frac{v}{s}$ (unit tangent vector)

$\vec{N}(t) = \frac{\vec{T}'}{||\vec{T}'||}$ (principle normal vector)

For $r(t) =< 2\,cos(t)\,, 2\,sin(t)\,, 3t >$ find tang. and normal comp. of accel.:

$$v(t) =< -2\sin(t)\,, 2\cos(t)\,, 3 >$$

$$s = \left||v|\right| = \sqrt{13}$$

$$a(t) = -2 < \cos(t)\,, \sin(t)\,, 0$$

$$v \cdot a = 0$$

$$v \times a = \begin{vmatrix} i & j & k \\ -2\sin(t) & 2\cos(t) & 3 \\ -2\cos(t) & -2\sin(t) & 0 \end{vmatrix} =$$

$$=< 6\sin(t)\,, -6\cos(t)\,, 4 >$$

Thus:

$$a_t = \frac{v \cdot a}{s} = 0$$

$$a_n = \frac{||v \times a||}{s} = \frac{\sqrt{52}}{\sqrt{13}} = 2$$