Summary 10: Tangent, Normal, and Binormal

Vector-valued function:

$$\vec{r}(t) =< x(t), y(t), z(t) >$$

Tangent vector:

$$\vec{r}'(t) =< x'(t), y'(t), z'(t) >$$

Unit tangent vector:

$$\vec{T}(t) = \frac{\vec{r}'}{||\vec{r}'||}$$

Unit or principal normal vector:

$$\vec{N}(t) = \frac{\vec{T}'}{||\vec{T}'||}$$

Binormal vector:

$$\vec{B}(t) = \vec{T} \times \vec{N}$$

$T \cdot T = 1, N \cdot N = 1, N \cdot N = 1$
$T \cdot N = 0, N \cdot B = 0, T \cdot B = 0$

Curvature:

$$\kappa = \frac{||\vec{T}'||}{||\vec{r}'||} = \frac{||\vec{r}' \times \vec{r}''||}{||\vec{r}'||^3}$$

$$r(t) =< 2\cos(t), 2\sin(t), 3t >$$

$$r'(t) =< -2\sin(t), 2\cos(t), 3 >$$

$$T = \frac{1}{\sqrt{13}} < -2\sin(t), 2\cos(t), 3 >$$

$$N =< -\cos(t), -\sin(t), 0 >$$

$$B = \begin{vmatrix} i & j & k \\ \frac{-2}{\sqrt{13}}\sin(t) & \frac{2}{\sqrt{13}}\cos(t) & \frac{3}{\sqrt{13}} \\ -\cos(t) & -\sin(t) & 0 \end{vmatrix}$$

$$=< \frac{3}{\sqrt{13}}\sin(t), \frac{-3}{\sqrt{13}}\cos(t), \frac{2}{\sqrt{13}} >$$

$$r' \times r'' = \begin{vmatrix} i & j & k \\ -2\sin(t) & 2cos(t) & 3 \\ -2\cos(t) & -2\sin(t) & 0 \end{vmatrix}$$

$$=< 6\sin(t), -6\cos(t), 4 >$$

$$\kappa = \frac{||r' \times r''||}{\sqrt{13}^3} = \frac{\sqrt{52}}{13\sqrt{13}} = \frac{2}{13}$$