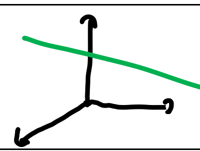
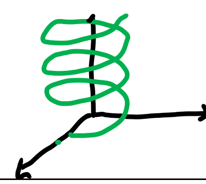
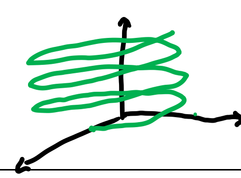
Summary 9: Space Curves

Vector-valued Functions aka Space Curves $\vec{r}(t) =< x(t), y(t), z(t) >$ where $x(t), y(t), z(t)$ are each functions of one variable. Usually you think of t as time and consider the graph as the path of a particle in space.	
Calculus with Space Curves Basic calculus operations can be done component-wise: Limits: $\lim_{t\to t_0} \vec{r}(t) =< \lim_{t\to t_0} x(t) , \lim_{t\to t_0} x(t) , \lim_{t\to t_0} x(t) >$ Derivatives: $\vec{r}'(t) =< x'(t), y'(t), z'(t) >$ Integrals: $\int \vec{r}(t)\,dt =< \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt >$	
Tangent and Unit Tangent If $\vec{r}(t)$ is a space curve, the *tangent* vector is $\vec{r}'(t)$ If $\vec{r}(t)$ is a space curve, the *unit tangent* is $\vec{T}(t) = {}^{\vec{r}'(t)}\big/_{\|\vec{r}'(t)\|}$	*If $\vec{r}(t) =< cos(t) , sin(t) , t >$, find $r'(0)$ and $T(\pi)$:* $\vec{r}'(t) =< -\sin(t) , \cos(t), 1 >$ and $\|r'(t)\| = \sqrt{2}$. Thus: $r'(0) =< 0,1,1 >$ and $T(\pi) = \frac{r'(\pi)}{\|r'(\pi)\|} = \frac{1}{\sqrt{2}} < 0,-1,1 >$
Length of a Space Curve Length L of a space curve $\vec{r}(t) =< x(t), y(t), z(t) >, a \le t \le b$, is: $L = \int_a^b \|\vec{r}'(t)\|\,dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$	*For $0 \le t \le Pi$, find the length of $\vec{r}(t) =< 2\,cos(t) , 2\,sin(t) , 3t >$:* $\vec{r}'(t) =< -2\sin(t) , 2\cos(t) , 3 >$ so $\|r'(t)\| = \sqrt{4+9} = \sqrt{13}$ and $\int_0^\pi \|r'(t)\|\,dt = \int_0^\pi \sqrt{13}\,dt = \pi\sqrt{13}$

Common Space Curves and their Graphs

Lines $\vec{r}(t) =< 1 + 2t, 3 - 4t, 2 + 2t >$	
Slinkys $\vec{r}(t) =< \cos(t) , \sin(t) , t >$	
Oval Slinkys $\vec{r}(t) =< a\cos(t) , b\sin(t), t >$	
Spirals $\vec{r}(t) =< t\cos(t) , t\sin(t) , t >$