Summary 5: Projection and Cross Product

Cross Product The cross product of $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ is: $\vec{a} \times \vec{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =< a_2 b_3 - a_3 b_2, -(a_1 b_3 - a_3 b_1), a_1 b_2 - a_2 b_1 >$	$< 1,0,1 > \times < 1,2,-3 > =$ $= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 1 \\ 1 & 2 & -3 \end{vmatrix} =$ $=< 0-2, -(-3-1), 2-0 >=< -2,4,2 >$
Properties of Cross Product $\vec{a} \times \vec{b}$ is a vector (as opposed to $\vec{a} \cdot \vec{b}$, which is a scalar) $\vec{a} \times \vec{a} = 0$ for any vector $\vec{a} \times \vec{b}$ is perpendicular to both vectors $\vec{a}$ and $\vec{b}$ $\left\|\vec{a} \times \vec{b}\right\| = \left\|\vec{a}\right\| \left\|\vec{b}\right\| \mid \sin(\theta) \mid$ $\vec{a} \times \vec{b} = 0$ iff $\vec{a}$ parallel to $\vec{b}$ (but it is easier to check if $\vec{a} = c\vec{b}$, c scalar)	
Lines Parametric equation of a line though point P with directional vector $\vec{v}$ is: $l(t) = P_0 + t\,\vec{v} = < p_1, p_2, p_3 > + t < v_1, v_2, v_3 > =$ $= < p_1 + tv_1, p_2 + tv_2, p_3 + t\,v_3 >$	Eq. of line through $P(1,3,-2)$ and $Q(4,2,1)$: dir. vector $\vec{v} = \overrightarrow{PQ} =$ $Q - P =< 3,-1,3 >$ so that $l(t) =$ $= (1,3,-2) + t < 3,-1,3 >=$ $=< 1+3t, 3-t, -2+3t >$
Properties A line $l(t)$ is a one-dimensional object inside $\boldsymbol{R}^n$ Two lines l_1 and l_2 are perpendicular if their directional vectors $\vec{v}_1$ and $\vec{v}_2$are perpendicular; they are parallel if their directional vectors $\vec{v}_1$ and $\vec{v}_2$ are parallel. Two lines l_1 and l_2 intersect if there are number *s* and *t* such that $l_1(t) = l_2(s)$	$l_1(t) =< 1-t, 2t, 3+2t >$ and $l_2(t) =< 3+2t, 1-3t, 1+4t>$ are perpendicular, because: $v_1 =< -1,2,2 >$ and $v_2 =< 2,-3,4 >$ so that $v_1 \cdot v_2 = 0$