Summary 4: Projection and Cross Product

Directional Angles The angle of a vector $v =< v_1, v_2, v_3 >$ and the respective axis is: x-axis: $\cos(\theta_x) = \frac{v_1}{\|\|v\|\|}$ y-axis: $\cos(\theta_y) = \frac{v_2}{\|\|v\|\|}$ z-axis: $\cos(\theta_z) = \frac{v_3}{\|\|v\|\|}$	x,z direct. angles of $< 1,2,3 >$: $\cos(\theta_x) = \frac{1}{\sqrt{14}}, \cos(\theta_z) = \frac{3}{\sqrt{14}}$
Projection The projection of vector $\vec{b}$ onto vector $\vec{a}$ is: $proj_{\vec{a}}(\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{\|\|\vec{a}\|\|^2}\vec{a}$ Its length is $comp_{\vec{b}}(\vec{a}) = \left\|proj_{\vec{a}}(\vec{b})\right\| = \frac{\|\vec{a}\cdot\vec{b}\|}{\|\|\vec{a}\|\|}$	$\vec{a} =< 1,0,1 >, \vec{b} =< 1,2,-3 >$: $proj_a(b) =$ $= \frac{<1,0,1>\cdot<1,2,-3>}{(\sqrt{2})^2} < 1,0,1 >=$ $= -\frac{2}{2} < 1,0,1 >=< -1,0,-1 >$
Cross Product The cross product of $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$ is: $\vec{a} \times \vec{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =< a_2b_3 - a_3b_2, -(a_1b_3 - a_3b_1), a_1b_2 - a_2b_1 >$	$< 1,0,1 > \times < 1,2,-3 > =$ $= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 1 \\ 1 & 2 & -3 \end{vmatrix} =$ $=< 0-2, -(-3-1), 2-0 >=< -2,4,2 >$
Properties of Cross Product $\vec{a} \times \vec{b}$ is a vector (as opposed to $\vec{a} \cdot \vec{b}$, which is a scalar) $\vec{a} \times \vec{a} = 0$ for any vector $\vec{a} \times \vec{b}$ is perpendicular to both vectors $\vec{a}$ and $\vec{b}$	