Summary 3: Vectors and Dot Product

Adding/ subtracting vectors algebraically: If $\vec{v} =< v_x, v_y, v_z >$ and $\vec{w} =< w_x, w_y, w_z >$, then • $\vec{v} + \vec{w} =< v_x, v_y, v_z > + < w_x, w_y, w_z >=$ $= < v_x + w_X, v_y + w_y, v_z + w_z >$ and • $\vec{v} + \vec{w} =< v_x, v_y, v_z > - < w_x, w_y, w_z >=$ $= < v_x - w_X, v_y - w_y, v_z - w_z >$	If $v =< 1,2,3 >$ and $w =< -2,3,-1 >$ then: $v + w =< -1,5,2 >$ $v - w =< 3,-1,4 >$
Basic Definitions: **Length** or **norm** of vector $\vec{v} =< v_x, v_y, v_z >$ is: $\|\|\vec{v}\|\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$	$\|\| < 1,3,-2 > \|\| = \sqrt{14}$
A **unit vector** $\vec{u}$ is a vector with norm 1: $\|\|\vec{u}\|\| = \sqrt{u_1^2 + u_2^2 + u_3^2} = 1$	$u =< \frac{\sqrt{3}}{2}, 0, \frac{1}{2} >$ is a unit vector
For any non-zero vector $\vec{v}$ there is a vector $\vec{u} = \frac{1}{\|\|\vec{v}\|\|}\vec{v}$ with the same direction and length	If $v =< 1,3,-2 >$, then $u = \frac{1}{\sqrt{14}} < 1,3,-2 >$ is unit vector
If $\vec{v} =< x, y >$ is a two dimensional vector in **polar coordinates**, i.e. with length l and angle θ, then $x = l\cos(\theta)$ and $y = l\sin(\theta)$	If v has length 2 and angle $\frac{\pi}{3}$, then $v =< 2\cos\left(\frac{\pi}{3}\right), 2\sin\left(\frac{\pi}{3}\right) >$
The vectors $\vec{\boldsymbol{i}} = < 1,0,0 >$, $\vec{\boldsymbol{j}} = < 0,1,0 >$, and $\vec{\boldsymbol{k}} = < 0,0,1 >$ are called the **unit basis vectors**. Every vector $\vec{v} =< v_1, v_2, v_3 >$ can be written as $v = v_1\vec{\boldsymbol{i}} + v_2\vec{\boldsymbol{j}} + v_3\vec{\boldsymbol{k}}$	$v =< 1,-3,2 >= \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$
The **Dot Product** **Definition:** The *dot product* between $\vec{v} =< v_1, v_2, v_3 >$ and $\vec{w} =< w_1, w_2, w_3 >$ is: $v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$.	$< 1,3,-2 > \cdot < 3,-2,-4 >$ $= 3 - 6 + 8 = 5$
Theorem: The dot product between two vectors is a scalar. It has the following properties: (i) $\vec{v} \cdot \vec{v} = \|\|\vec{v}\|\|^2$, (ii) $\vec{v} \cdot (\vec{w} + \vec{z}) = \vec{v} \cdot \vec{w} + \vec{v} \cdot \vec{z}$, (iii) $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$	
Theorem: The dot product between two vectors gives, essentially, the (cosine of) the angle between them. Specifically: $\frac{\vec{v} \cdot \vec{w}}{\|\|\vec{v}\|\|\|\|\vec{w}\|\|} = \cos(\theta)$.	Angle between $v =< -1,2,2 >$ and $w =< 3,-2,-4 >$ is: $\cos(\theta) = \frac{<1,2,2> \cdot <3,2,1>}{\sqrt{9}\sqrt{14}} = \frac{9}{\sqrt{126}}$
Corollary: Two vectors $\vec{v}$ and $\vec{w}$ are *perpendicular* iff $\vec{v} \cdot \vec{w} = 0$	$< 1,-4,0 >$ and <8,2,9> are perpendicular because $< 1,-4,0 > \cdot < 8,2,9 >= 0$