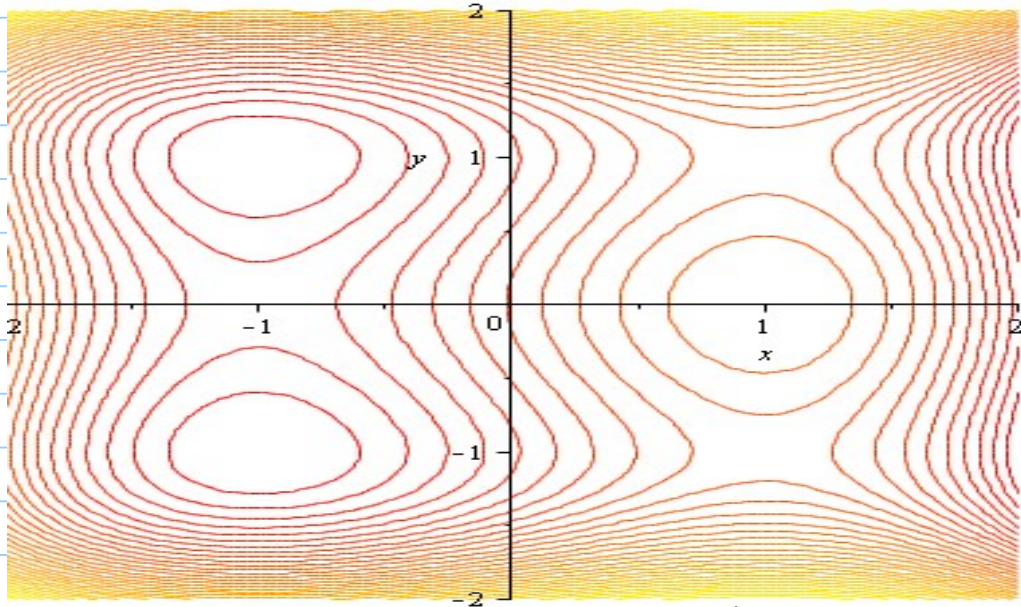


Calc 3 - Assignment # 16

Note Title

10/19/2011

- ① Use the contour plot of $f(x,y) = 3x - x^3 - 2y^2 + y^4$ to identify local min, max, or saddle points if any.



(Hint: there are six critical points)

- ② Find all max, min, and saddle points for

a) $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

b) $g(x,y) = x^3y + 12x^2 - 8y$

- ③ Find the local max, min, and saddle points,

if any, for $f(x,y) = x^4 + y^4 - 4xy + 1$.

Visualize your answer by drawing the surface and/or contour plot in Mathematica!

(Note: there are three critical points)

④ Use Maple to draw the surfaces / contour plots for the following functions and guess any

max, min, and saddle points.

a) $f(x, y) = x^2 + y^2 + x^2 y^2$

b) $g(x, y) = x^4 - 5x^2 + y^2 + 3x + 2$

⑤ Continuous functions of one variable can not have two local max without having a local min. For two-variable functions this is different:

Show that

$$f(x, y) = -(x^2 - 1)^2 - (x^2 y - x - 1)^2$$

has only 2 critical points, both of which are max. Use Mathematica to visualize the function.

⑥ Find the point on the plane $x - y + z = 4$ closest to $(1, 2, 3)$.

⑦ Find three positive numbers whose sum is 100 and whose product is max.

⑧ Find the rectangular box with largest volume and total surface area of 64 cm^2 .

8) Find the directional derivative of $f(x,y)$ at the given point in the direction indicated by θ :

a) $f(x,y) = x^2 y^3 - y^4$, $P(2,1)$, $\theta = \pi/4$

b) $f(x,y) = x \sin(xy)$, $P(2,0)$, $\theta = \pi/3$

9) Find the gradient of f at the point

a) $f(x,y) = \frac{y^2}{x}$, $P(1,2)$

b) $f(x,y,z) = \sqrt{x+yz}$, $P(1,3,1)$

10) Find the directional derivative $D_{\vec{v}}(f)$:

a) $f(x,y) = \ln(x^2 + y^2)$, $\vec{v} = \langle -1, 2 \rangle$

b) $f(x,y,z) = (x+2y+3z)^{3/2}$, $\vec{v} = 2\vec{j} - \vec{k}$

⑤ Find max. rate of change and its direction:

a) $f(x,y) = \frac{y^2}{x}$ at $(2,4)$

b) $f(x,y,z) = \frac{x+y}{z}$ at $(1,1,-1)$

⑥ The depth of a lake is given by

$$d = 200 + 0.02x^2 - 0.001y^3 \text{ measured in feet.}$$

A fisherman in a small boat starts at $(80,60)$ and moves toward $(0,0)$. Is the water under the boat getting deeper or shallower?

⑦ The temperature at a point (x,y,z) is

$$T(x,y,z) = 200 e^{-x^2 - 3y^2 - 9z^2}$$

a) Find the rate of change of T at $P(2,-1,2)$

in the direction toward $(3,-3,7)$

b) In which direction does T increase the fastest?

Find the max. rate of increase at P