Calc 2 Practice Exam Supplement

Here are a few supplementary questions involving mass, moments, and center of gravity. Any of these questions could appear on the exam 2 (or not).

1. Define the mass m, the moments M_x, M_y, and the center of gravity $(\bar{x}, \bar{y})$

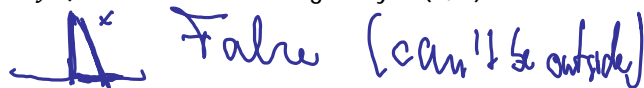$$m = \iint_D \rho(x,y) \, dA \qquad (\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

$$M_x = \iint_D y \, \rho(x,y) \, dA \qquad M_y = \iint_D x \, \rho(x,y) \, dA$$

2. True/False
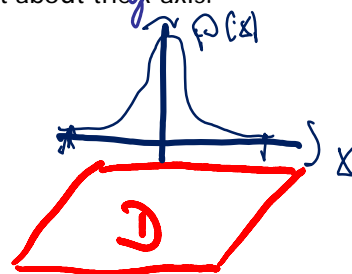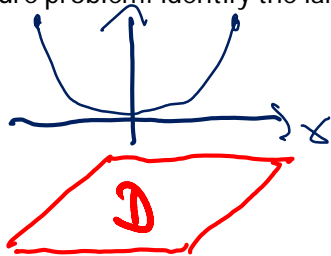   a. If D is a lamina of uniform density in form of a rectangle from (0,0) to (2,4), then the center of gravity $(\bar{x}, \bar{y}) = (1,2)$ True

   b. If D is a lamina bounded by the x-axis, the y-axis, and the line from (0, 2) to (1, 0) with density function $\rho(x,y) = x^2 \sin(y) * \cos(x^2 + y^2)$ then the center of gravity is (1, 2) False (can't be outside)

3. Picture problem: identify the lamina with the larger moment about the y-axis:

   X more mass far away from y-axis

4. Find the mass and center of gravity of the lamina that occupies the region D and has the given density function:
   a. $D = \{(x,y) : 0 \le x \le 2, -1 \le y \le 1\}$ and $\rho(x,y) = xy^2$

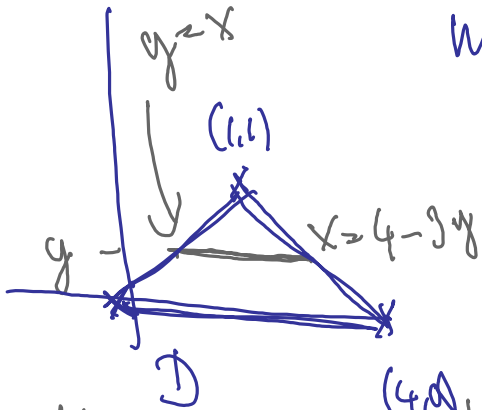$$m = \iint x y^2 \, dy \, dx = \frac{4}{3}$$

$$M_x = \iint x y^3 \, dy \, dx = 0 \qquad (\bar{x}, \bar{y}) = \left(\frac{4}{3}, 0 \right)$$

$$M_y = \iint x^2 y^2 \, dy \, dx = \frac{16}{9}$$

b. D is the triangular region with vertices (0,0), (1,1), (4,0) and $\rho(x,y) = x$

$y=x$

(1,1)

$x = 4 - 3y$

$y-$

D

$$M = \iint_D x\, dA = \int_0^1 \int_y^{4-3y} x\, dx\,dy = \frac{10}{3}$$

$$M_x = \int_0^1 \int_y^{4-3y} y \times x\, dx\,dy =] \quad M_y = \int_0^1 \int_y^{4-3y} x^2\, dx\,dy = 7$$

$M \approx \frac{1}{3}$, $g-0 = -\frac{1}{3}(x-4)$, $y = -\frac{1}{3}x + \frac{4}{3}$

$(4,0)$

$\Rightarrow (\bar{x}, \bar{y}) = \left(\frac{4}{10}, \frac{3}{10}\right)$

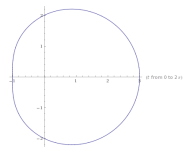c. D is bounded by the parabola $x = y^2$ and the line $y = x - 2$, and $\rho(x,y) = 3$

$x = y^2$

$y = x - 2$
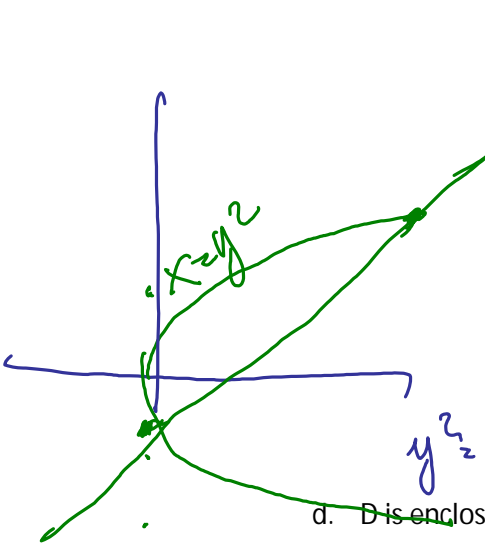
$$M = \int_{-1}^{2} \int_{y^2}^{y+2} 3\, dx\,dy$$

$$M_x = \int_{-1}^{2} \int_{y^2}^{y+2} 3y\, dx\,dy =$$

$$M_y = \int_{-1}^{2} \int_{y^2}^{y+2} 3x\, dx\,dy =$$

$y^2 = y + 2$ or $y = 2, -1$

d. D is enclosed by the cardioid $r = 1 + \cos(\theta)$ and $\rho(x,y) = \sqrt{x^2 + y^2}$

$$M = \int_0^{2\pi} \int_0^{1+\cos\theta} \sqrt{r^2}\, r\, dr\, d\theta = \frac{5\pi}{3}$$

$$M_x = \int_0^{2\pi} \int_0^{1+\cos\theta} r\sin\theta\sqrt{r^2}\, r\, dr\, d\theta = 0$$

$$M_y = \int_0^{2\pi} \int_0^{1+\cos\theta} r\cos\theta\sqrt{r^2}\, r\, dr\, d\theta = \frac{7\pi}{4}$$

$(\bar{x}, \bar{y}) = \left(\frac{\frac{7\pi}{4}}{\frac{5\pi}{3}}, 0\right)$

$= \left(\frac{7\pi}{4} \cdot \frac{3}{5\pi}, 0\right) =$

$= \left(\frac{21}{20}, 0\right)$