

Math 2511 – Calc III Practice Exam 1

This is a practice exam. The actual exam consists of questions of the type found in this practice exam, but will be shorter. If you have questions do not hesitate to send me email.

- Definitions:** Please state in your own words the meaning of the following terms:
 - Vector
 - Dot product, cross product
 - Angle between two vectors
 - Unit vector
 - Line, plane
 - Projection of vector a onto vector b
 - Distance formulas between planes and points, lines and points, planes and lines, planes and planes, lines and lines
 - Tangent vector to a curve
 - Unit tangent vector to a curve
 - Unit normal vector to a curve
 - Binormal vector
 - Curvature
 - Length of a curve
 - Integral of a curve
 - Slinky, spirals
- True/False questions:**
 - $u \cdot u = \|u\|^2$
 - $\langle 1, 3, 2 \rangle$ and $\langle -4, -2, 5 \rangle$ are perpendicular
 - $\langle 1, 3, -2 \rangle$ and $\langle 2, 6, 4 \rangle$ are parallel
 - $v \cdot w = -w \cdot v$
 - $\frac{d}{dt} \|r(t)\| = \left\| \frac{d}{dt} r(t) \right\|$
 - $\frac{d}{dt} p(t) \times r(t) = p'(t) \times r'(t)$
 - $r(t) = \langle \sqrt{t} + 2, 3 - \sqrt[3]{t}, \sqrt[4]{t} \rangle$ is the equation of a line
 - If $\|r(t)\| \equiv 1$ then $r(t) \times r'(t) = 0$
 - The planes $x + 3y + 2z = 5$ and $4x + 2y - 5z = 0$ are perpendicular
 - The distance between $x - y + z = 2$ and $x + y + z = 1$ is zero
- Vectors:** Suppose $u = \langle 7, -2, 3 \rangle$, $v = \langle -1, 4, 5 \rangle$, and $w = \langle -2, 1, -3 \rangle$
 - Are u and v orthogonal, parallel, or neither?
 - Find graphically and algebraically $2u + 3v$ and $u - v$
 - Find the angle between v and w
 - Find $u \cdot v$ (dot product), $u \times v$ (cross product), $u \cdot (v \times w)$, and $\|u\|$
 - Find the projection of w onto u and the projection of u onto w
- Lines and Planes**

- Find the equation of the plane spanned by $\langle 1, 3, -2 \rangle$ and $\langle 2, 1, 2 \rangle$ through the point $P(1, 2, 3)$
- Find the equation of the plane through $P(1, 2, 3)$, $Q(1, -1, 1)$, and $R(3, 2, 1)$
- Find the equation of the plane parallel to $x - y + z = 2$ through $P(0, 2, 0)$
- Find the equation of the line through $P(1, 2, 3)$ and $Q(1, -1, 1)$
- Do the plane $x - y + z = 2$ and the line $l(t) = \langle 1 + t, 2t, 1 - 5t \rangle$ intersect? If so, where?

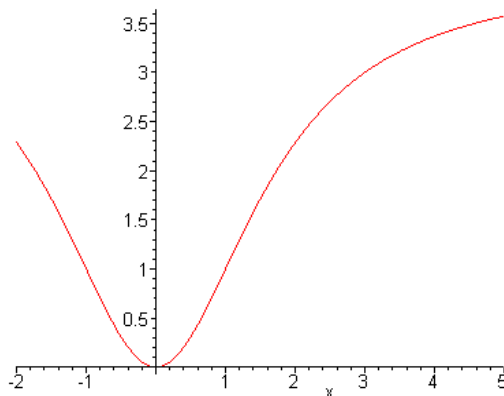
5. **Distances**

- Find the distance between the line $x - y = 2$ and $P(1, 2)$
- Find the distance between the plane $x + y + z = 1$ and the point $P(1, 2, 3)$
- Find the distance between the planes $x - y + z = 2$ and $2x - 2y + 2z = 5$

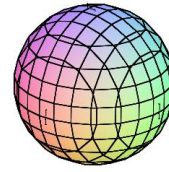
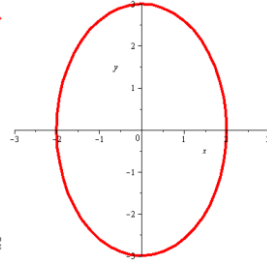
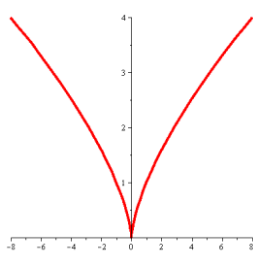
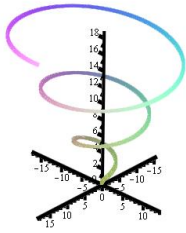
6. **Vector valued functions:**

- Find $r'(t)$ if $r(t) = \langle 6t, -7t^2, t^3 \rangle$
- Find $r'(t)$ if $r(t) = \langle a \cos^3(t), a \sin^3(t), t \sin(t) \rangle$
- If $r(t) = \langle 4t, t^2, t^3 \rangle$, find $r'(t)$, $r''(t)$, $\frac{d}{dt} \|r(t)\|$
- If $r(t) = \langle e^t, 3t^3, \frac{3}{6t} \rangle$ some curve, find $\int_1^2 r(t) dt$
- If $r(t) = \langle t, \frac{1}{t} \rangle$, find $T(t)$, $N(t)$, and $B(t)$
- Repeat (e) for $r(t) = \langle e^t \cos(t), e^t \sin(t) \rangle$ for $t = \frac{\pi}{2}$
- If $r(t) = \langle 3 - 3t, 4t \rangle$, find the arc length of the curve between 0 and 1
- If $r(t) = \langle 4t, 3 \cos(t), 3 \sin(t) \rangle$, find the arc length of the curve between 0 and $\frac{\pi}{2}$
- Find the curvature of $r(t) = \langle t, 3t^2, \frac{t^2}{2} \rangle$

8. **Picture:** Sketch the circle that fits the graph below the best at the points $x = 0$ and $x = 3$. At which of the two points is the curvature smaller?

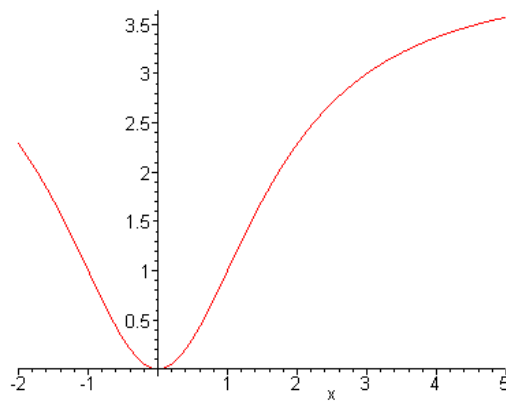


9. **Picture:** Match the following functions to their corresponding plots.



$$r(t) = \langle t^3, t^2 \rangle \quad x^2 + y^2 + z^2 = 1 \quad r(t) = \langle t \cos(t), t \sin(t), t \rangle \quad r(t) = \langle 2 \sin(t), 3 \cos(t) \rangle$$

10. **Picture:** The graph below shows a vector-valued function. Sketch the unit tangent, unit normal, acceleration, tangential and normal components of the acceleration for $t = 3$.



11. **Prove** the following facts:

- Show that $u \times v = -(v \times u)$
- Show that $u \cdot (v \times u) = 0$
- Show that if $y = f(x)$ is a function that is twice continuously differentiable, then the curvature of f at a point x is
$$K = \frac{|f''(x)|}{(1 + [f'(x)]^2)^{3/2}}$$
- Prove that the curvature of a line in space is zero.