Calc 3, Assignment 30

1. Please state:
   a) What is Green's Theorem?
   b) What is Gauss' Theorem? What is its alternate name?
   c) For what type of surface can you apply the Divergence theorem?

a) Find the following **surface areas**:
   a) of the plane $z = 2 - x - y$ above the rectangle $0 \le x \le 2$ and $0 \le y \le 3$
   b) of the cylinder $z = 9 - y^2$ above the triangle bounded by $y = x$, $y = -x$, and $y = 3$
   c) of the surface $z = 16 - x^2 - y^2$ above the circle $x^2 + y^2 \le 9$

3. Evaluate the following **3D volume integrals:**
   a) $\iiint_B xyz^2\,dV$, where B is the rectangular box given by $\{0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$
   b) $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$
   c) $\iiint_E \sqrt{x^2 + z^2}\,dV$, where E is the region bounded by $y = x^2 + z^2$ and $y = 4$

4. Find the following line integrals. You may use Maple or Wolfram Alpha to help you out.
   a) Find the surface integral $\iint_S x - 2y + z dS$, where S is the surface $z = 10 - 2x + 2y$ such that x is between 0 and 2 and y is between 0 and 4.
   b) $\iint_S (x+z) dS$ where S is the first-octant portion of the cylinder $y^2 + z^2 = 9$ between x = 0 and x = 4
   c) The flux of the vector field $\vec{F}(x, y, z) = \langle x, y, z \rangle$, where S is the portion of the surface $z = 10 - 2x - 2y$ between the coordinate planes.
   d) The flux of the vector field $F(x, y, z) = \langle x, y, z \rangle$ through the surface given by potion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane. Note that this surface is *not* closed.
   e) Evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x, y, z) = \langle z^2, x^2, y^2 \rangle$ and S is the closed surface given by $z = 4 - x^2 - y^2$ above the xy-plane together with the "lid" $z = 0$.
   f) Evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x, y, z) = \langle x, y, z \rangle$ and S is $x^2 + y^2 + z^2 = 4$