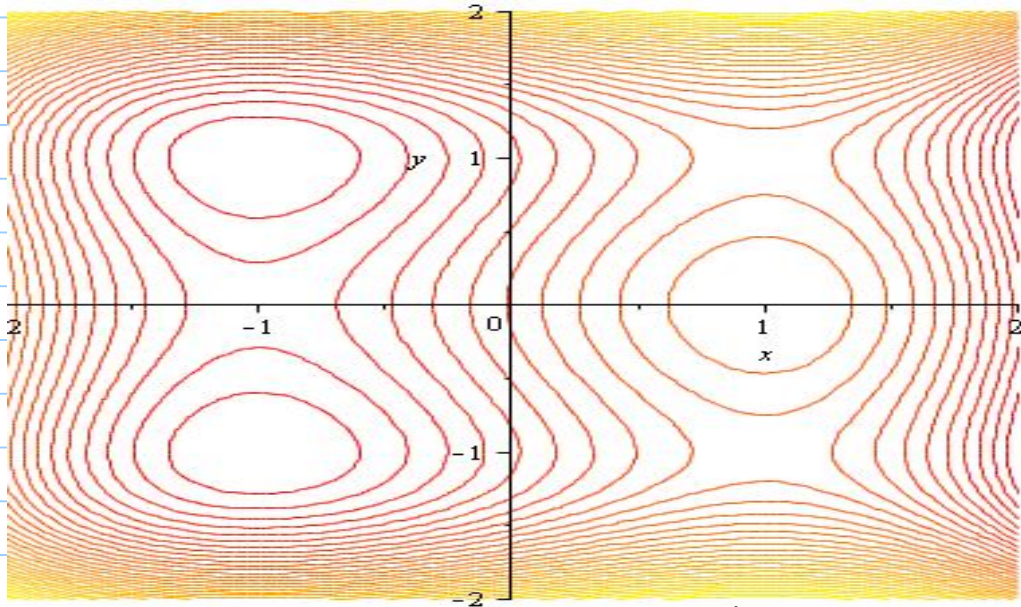


## Calc 3 - Assignment # 16

- ① Use the contour plot of  $f(x,y) = 3x - x^3 - 2y^2 + y^4$  to identify local min, max, or saddle points if any.



(Hint: there are six critical points)

- ② Find all max, min, and saddle points for

a)  $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

b)  $g(x,y) = x^3y + 12x^2 - 8y$

- ③ Find the local max, min, and saddle points,

if any, for  $f(x,y) = x^4 + y^4 - 4xy + 1$ .

Visualize your answer by drawing the surface and/or contour plot in Maple.

(Note: there are three critical points)

④ Use Maple to draw the surfaces / contour plots for the following functions and guess any

max, min, and saddle points.

a)  $f(x, y) = x^2 + y^2 + x^2 y^2$

b)  $g(x, y) = x^4 - 5x^2 + y^2 + 3x + 2$

⑤ Continuous functions of one variable can not have two local max without having a local min. For two-variable functions this is different:

Show that

$$f(x, y) = -(x^2 - 1)^2 - (x^2 y - x - 1)^2$$

has only 2 critical points, both of which are max. Use Maple to visualize the function.

⑥ Find the point on the plane  $x - y + z = 4$  closest to  $(1, 2, 3)$ .

⑦ Find three positive numbers whose sum is 100 and whose product is max.

⑧ Find the rectangular box with largest volume and total surface area of  $64 \text{ cm}^2$ .

⑤ Find max. rate of change and its direction:

a)  $f(x,y) = \frac{y^2}{x}$  at  $(2,4)$

b)  $f(x,y,z) = \frac{x+y}{z}$  at  $(1,1,-1)$

⑥ The depth of a lake is given by

$$d = 200 + 0.02x^2 - 0.001y^3 \text{ measured in feet.}$$

A fisherman in a small boat starts at  $(90,60)$  and moves toward  $(0,0)$ . Is the water under the boat getting deeper or shallower?

⑦ The temperature at a point  $(x,y,z)$  is

$$T(x,y,z) = 200 e^{-x^2 - 3y^2 - 9z^2}$$

a) Find the rate of change of  $T$  at  $P(2,-1,2)$

in the direction toward  $(3,-3,7)$

b) In which direction does  $T$  increase the fastest?

Find the max. rate of increase at  $P$