

## Calc 3 - Assignment 15

① If  $f(x,y) = \sqrt{4-x^2-4y^2}$ , find  $f_x(1,0)$  and  $f_y(1,0)$  and interpret as slopes.

② Verify that  $u_{xy} = u_{yx}$  for  $u(x,y) = x \sin(x+2y)$

③ Which is a solution to Laplace's PDE  $u_{xx} + u_{yy} = 0$

a)  $u = x^2 + y^2$

b)  $u = x^2 - y^2$

c)  $u = \sin(x) \cosh(y) + \cos(x) \sinh(y)$

d)  $u = e^{-x} \cos(y) - e^{-y} \cos(x)$

④ Show that  $u = \frac{1}{a^2} t^2 - x^2$  solves the Wave Equation  $u_{tt} = a^2 u_{xx}$  ..

⑤ How many  $n$ -th order partial derivatives does a function  $f(x,y)$  have?

⑥ Find equation of the tangent plane to the given surface at the specified point:

a)  $z = y \ln(x)$  at  $(1,4,0)$

b)  $z = \sqrt{xy}$  at  $(1,1,1)$

7) Graph the surface and its tangent plane at the given point using Maple:

$$z = x^2 + xy + 3y^2 \text{ at } (1, 1, 5)$$

8) Find the directional derivative of  $f(x, y)$  at the given point in the direction indicated by  $\theta$ :

a)  $f(x, y) = x^2 y^3 - y^4$ ,  $P(2, 1)$ ,  $\theta = \pi/4$

b)  $f(x, y) = x \sin(xy)$ ,  $P(2, 0)$ ,  $\theta = \pi/3$

9) Find the gradient of  $f$  at the point  $P$ :

a)  $f(x, y) = \frac{y^2}{x}$ ,  $P(1, 2)$

b)  $f(x, y, z) = \sqrt{x + yz}$ ,  $P(1, 3, 1)$

10) Find the directional derivative  $D_{\vec{v}}(f)$ :

a)  $f(x, y) = \ln(x^2 + y^2)$ ,  $\vec{v} = \langle -1, 2 \rangle$

b)  $f(x, y, z) = (x + 2y + 3z)^{3/2}$ ,  $\vec{v} = 2\vec{j} - \vec{k}$