Calc 3 - Assignment #9

(1) Find the curvature:

a) $\vec{r}(t) = \langle 3t, 4\sin(t), 4\cos(t) \rangle$

b) $r(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle$, $0 \le t \le 1$

c) $r(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$, $0 \le t \le 1$

d) $\vec{r}(t) = \langle t^2, t \rangle$

e) $\vec{r}(t) = \langle e^t \cos(t), e^t \sin(t), t \rangle$

(2) Show that the circular helix $\vec{r}(t) = \langle a\cos(t), a\sin(t), bt \rangle$, $a > 0, b > 0$, has constant curvature.

(3) There is another formula for the curvature that is often easier:

$$\kappa = \frac{\|T'(t)\|}{\|r'(t)\|} = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|r'(t)\|^3}$$

Use this formula to redo #1 (b)-(e)

(4) Find a formula for the curvature of a function $y = f(x)$. Hint: write the function in parametric form, then embed in $\mathbb{R}^3$. Then compute the curvature using cross products.

(5) Which of the following curves is the original function $f(x)$, which is its curvature as a function: