Calc I: Assignment #8

① Sketch or describe the following curves:

a) $r(t) = \langle \cos(t), \sin(t) \rangle$

b) $r(t) = \langle t, \sin(t) \rangle$

c) $r(t) = \langle t, \cos(t), \sin(t) \rangle$

d) $r(t) = \langle 1+t, 3t, -t \rangle$

② Find the following limits:

a) $\lim_{t \to 0} \left\langle \frac{e^t - 1}{t}, \frac{\sqrt{1+t} - 1}{t}, \frac{3}{1+t} \right\rangle$

b) $\lim_{t \to \infty} \left\langle \arctan(t), e^{-2t}, \frac{\ln(t)}{t} \right\rangle$

③ Find the derivatives of:

a) $\vec{r}(t) = \langle t\sin(t), \tan(t), t\cos(2t) \rangle$

b) $\vec{r}(t) = e^{t^2} i - j + \ln(1+3t) k$

④ Find $\int_1^2 t^2 i + t\sqrt{t-1}\, j + t\sin(\pi t) k \, dt$

⑤ Two particles travel along the space curves

$r_1(t) = \langle t, t^2, t^3 \rangle$, $r_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$

Do they collide? Do their paths intersect?

(6) Use Maple's spacecurve command (or Wolfram Alpha) to plot in $\mathbb{R}^3$:

a) $\langle (1+\cos(16t))\cos(t), (1+\cos(16t))\sin(t), 1+\cos(16t) \rangle$

b) $\langle \cos(t)\sin(2t), \sin(t)\sin(2t), \cos(2t) \rangle$

c) $\langle t, t\sin(t), t\cos(t) \rangle$

(7) Prove the product rule for vector valued functions $\vec{u}(t), \vec{v}(t)$ in $\mathbb{R}^2$.

Valentine's Special: Use Maple's "plot" command (or Wolfram Alpha) to draw the 2D parametric curve given by

$\vec{r}(t) = \langle \cos(t)\sin(t)\ln(|t|), |t|^{0.3}\sqrt{\cos(t)} \rangle$ for $-1 \le t \le 1$.

Hint: plot([cos(t), sin(t), t=0..2*PI]) should work in Maple

parametric plot [sin(t), cos(t)] as t=0 works in WA,

as examples for 2D functions.

Also try to plot the 2D curve:

$\vec{r}(t) = \langle 3.9\cdot \sin^3(t), 3\cos(t) - 1.2\cos(2t) - 0.6\cos(3t) - 0.2\cos(4t) \rangle$

for $0 \le t \le 2\pi$ Which one do you like better?

(8) Find the arc length of the slinky

$r(t) = \langle \cos(t), \sin(t), t \rangle$, $0 \le t \le 2\pi$