

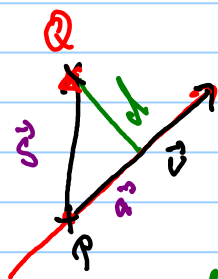
Calc 3 HW: Distances + Intersections

① Find the following distances:

- between $P(2, 5, 5)$ and the plane $x - 2y - 2z = 2$
- between $P(3, -2, 1)$ and $4x - 6y - z = 5$
- between $P(1, 2, 3)$ and $l(t) = \langle 2, 1, -3 \rangle + t \langle 2, 2, -1 \rangle$
- between planes $x + 2y - z = 1$ and $3x + 6y - 2z = 5$
- between planes $3x + 6y - 2z = 4$ and $x - 2y + z = 3$
- between $P(-3, 4)$ and line $y = 2x - 5$

② Find line of intersection between planes $x + y + z = 1$ and $x - 2y + 3z = 1$

③ Show that the distance of Q to a line $l(t) = P + t\vec{v}$ is $d = \frac{\|\vec{v} \times \vec{PQ}\|}{\|\vec{v}\|}$



Hint: d is the distance, or wait
 $\vec{a} = \text{proj}_{\vec{v}}(\vec{PQ})$, $\vec{s} = \vec{PQ}$

Use the Fact: $\vec{a} \times (\vec{s} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{s} - (\vec{a} \cdot \vec{s})\vec{c}$

Use this formula to find the distance between

- $P(1, 2, 3)$ and $l(t) = \langle -1, 1, -1 \rangle + t \langle 2, 3, 1 \rangle$
- $P(3, 0, 4)$ and $l(t) = \langle 2, -2, 5 \rangle + t \langle 1, 3, -1 \rangle$

④ Review the formula for the distance between $P(x_0, y_0, z_0)$ and $ax + by + cz + d = 0$ and explain why the distance is zero if P is on the plane.

⑤ How do you find the distance between two lines? Hint: the smallest distance is along a vector perp. to both lines. Now project a vector PQ with P on line 1 and Q on line 2 onto that vector. Ex: Find the distance between $l_1(t) = \langle 2, 0, 1 \rangle + t \langle 1, 1, 0 \rangle$ and $l_2(t) = \langle 0, 1, 1 \rangle + t \langle -1, 1, -1 \rangle$ (Answer is $\frac{3}{\sqrt{2}}$)