Panel 1

Final: Take-Home

out it Sat.

due on Wed, 5/7.

in class, mailbox, or email

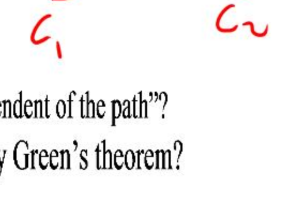
Panel 2

Definitions and Concepts:

- Vector, Angle between two vectors, Unit vector, dot product, cross product, projections
- Tangent vector to a curve, normal vector to a curve, binormal vector
- Velocity, speed, and acceleration, tangential and normal component of acceleration, curvature

3D vectors

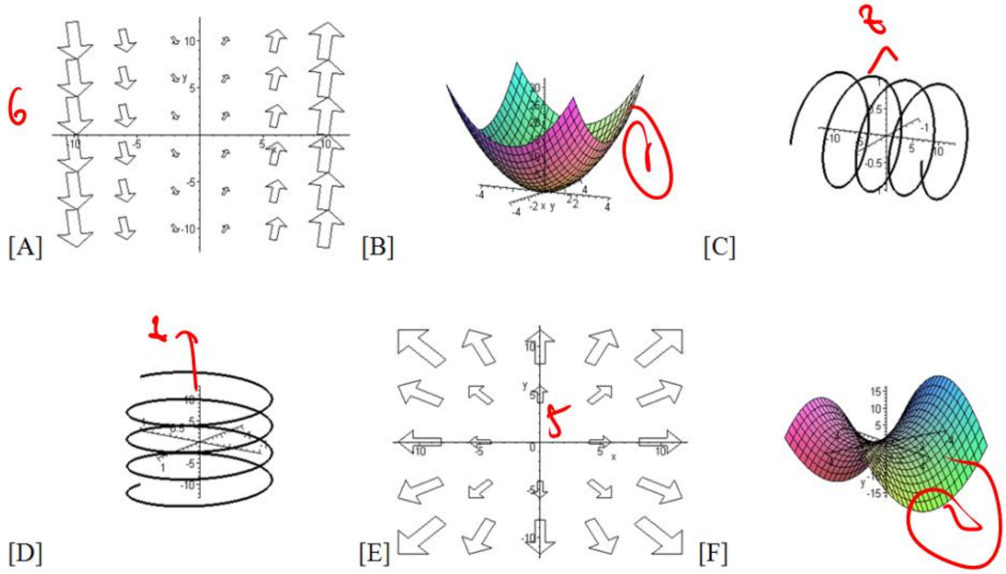
- Lines, planes, and distances
- Limit of a function $z = f(x, y)$
- Continuity of a function $z = f(x, y)$
- partial derivative of a function f(x,y); directional derivative
- gradient and its properties, curl and divergence
- the procedure to find relative extrema of a function f(x, y)
- double and triple integrals, including polar coordinates
- What does it mean when a "line integral of a vector field F is independent of the path"?
- What is Green's Theorem? For what type of curve can you apply Green's theorem?
- What is Gauss' Theorem? For what type of surface can you apply the Divergence theorem?
- What is Stoke's Theorem? For what type of curve can you apply Stoke's theorem?



$$\int_{C_1} F\,dr = \int_{C_2} F\,dr$$

Panel 3

problem: Match the following pictures with the algebraic expressions below.



(1) $f(x,y)=x^2+y^2$ (2) $f(x,y)=x^2-y^2$ (3) $r(t)=\langle\cos(t),\sin(t),t\rangle$

(4) $r(t)=\langle\cos(t),t,\sin(t)\rangle$ (5) $F(x,y)=\langle x,y\rangle$ (6) $F(x,y)=\langle 1,x\rangle$

Panel 4

Vectors: Suppose $u=<7,-2,3>$, $v=<-1,4,5>$, and $w=<-2,1,-3>$

- Are u and v orthogonal, parallel, or neither?
- Find the (cos of the) angle between v and w $w\cdot(v\times w)=0$
- Find $u\cdot v$ (dot product), $u\times v$ (cross product), $u\cdot(v\times w)$, and $\|u\|$
- Find the projection of w onto u and the projection of u onto w

$$\cos(\theta)=\frac{v\cdot w}{\|v\|\|w\|}$$

$$proj_{\vec{v}}(\vec{u})=\left(\frac{u\,v}{\|v\|^2}\right)v$$

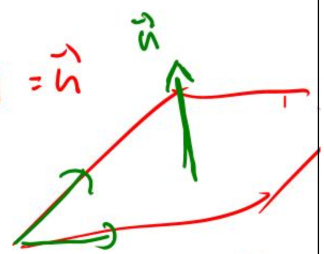
$$proj_{\vec{u}}(\vec{v})=\left(\frac{u\cdot v}{\|u\|^2}\right)u$$

Panel 5

Lines and Planes

- Find the equation of the plane spanned by $<1,3,-2>$ and $<2,1,2>$ through the point $P(1,2,3)$
- Find the equation of the plane through $P(1,2,3)$, $Q(1,-1,1)$, and $R(3,2,1)$
- Find the equation of the plane parallel to $x-y+z=2$ through $P(0,2,0)$
- Find the equation of the line through $P(1,2,3)$ and $Q(1,-1,1)$
- Some distance questions

Plane: $ax+by+cz=d$, $\langle a,b,c\rangle = \vec{n}$



$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 6+2, -(2+4), (-6)\rangle = \langle 8,-6,-5\rangle$$

$8x-6y-5z=d$

$8\cdot 1-6\cdot 2-5\cdot 3=d$

Panel 6

$r(t) = A + t(B-A)\ ,\ t\in[0,1]$

$(1,2,3)+t((1,-1,1)-(1,2,3))=$

$(1,2,3)+t(0,-3,-2)\ ,\ t\in[0,1]$

$r(t) = P_0 + t\vec{v}$

Find line ~~parallel~~ perp to $\langle 1,2,3\rangle + t\langle -1,2,1\rangle$

through $P_0=(0,0,1)$

$r(t) = P_0 + (v)t = (0,0,1)+\langle -1,2,1\rangle t$

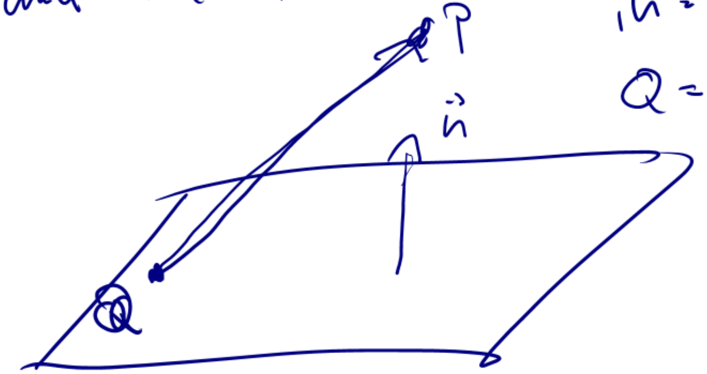
$r(t) = (0,0,1)+t\cdot\langle +1,0,+1\rangle$

Panel 7

Distance between $5x+4y+3z=2$ and $P(1,1,1)$.

$\vec{n} = \langle 5,4,3 \rangle$

$Q = \langle 0, \frac{1}{2}, 0 \rangle$



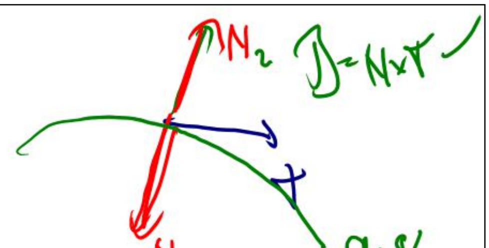
$PQ = \langle -1, -\frac{1}{2}, -1 \rangle$

$$d = \|proj_{\vec{n}} PQ\| = \frac{|\langle -1,-\frac{1}{2},-1\rangle \cdot \langle 5,4,3\rangle|}{\sqrt{50}}$$

Panel 8

Vector valued functions:

- If $r(t) = < 4t, t^2, t^3 >$, find $r'(t)$, $r''(t)$, $\frac{d}{dt}\|r(t)\|$
- If $r(t) = < e^t, 3t^3, \frac{3}{6t} >$ some curve, find $\int_1^2 r(t)dt$
- If $r(t) = < t, \frac{1}{t} >$, find $T(t)$, $N(t)$, a_t and a_n
- Repeat the previous exercise for $r(t) = < e^t\cos(t), e^t\sin(t) >$ for $t = \frac{\pi}{2}$



$B = N \times T$

$a_t = \frac{a \cdot v}{s}$

$a_N = \frac{\|a \times v\|}{s}$

$r(t) = \langle t, \frac{1}{t} \rangle$

$$T = \frac{r'}{\|r'\|} = \frac{\langle 1, -1/t^2 \rangle}{\sqrt{1+1/t^4}} = \frac{\langle 1, -1/t^2 \rangle t^2}{\sqrt{t^4+1}} = \frac{\langle t^2, -1 \rangle}{\sqrt{t^4+1}}$$

$$N = \frac{T'}{\|T'\|} = \frac{\langle 1, t^2 \rangle}{\sqrt{t^4+1}} \text{ or } -\frac{\langle 1, t^2 \rangle}{\sqrt{t^4+1}}$$

Panel 9

Motion in space:

- If $r(t) = < t, 3t^2, \frac{t^2}{2} >$ represents the position vector of a particle, find the velocity, speed, and acceleration, as well as tangential ~~and normal~~ components of the acceleration
- A baseball is hit 3 feet above ground at 100 feet per second and at an angle of Pi/4 with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10-foot high fence located 300 feet from home base?

$v(0) = \langle \frac{100}{\sqrt{2}}, \frac{100}{\sqrt{2}} \rangle = \langle c, d \rangle$

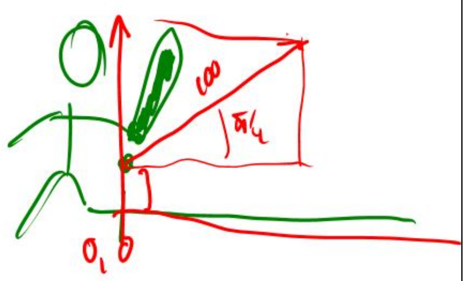
$\vec{a} = \langle 0, -g \rangle$

$\vec{s}(t) = \langle \frac{100}{\sqrt{2}} t, -\frac{1}{2} g t^2 + \frac{100}{\sqrt{2}} t + 3 \rangle$

$\vec{v} = \langle c, -gt + d \rangle$

$\vec{s} = \langle ct, -\frac{1}{2} g t^2 + dt + e$

$s(0) = \langle 0, 3 \rangle = \langle 0, e \rangle$



Panel 10

Limits and Continuity: Determine the following limits as $(x,y) \to (0,0)$, if they exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2+1} = 1 \qquad \lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2} = \text{NaN} \qquad \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \text{ DNE}$$

$\boxed{\text{plot}}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2} = 0 \qquad \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

$x = 0$

$y = 0$

$x = y$

$x = y^2$

$y = x^2$

[illegible]