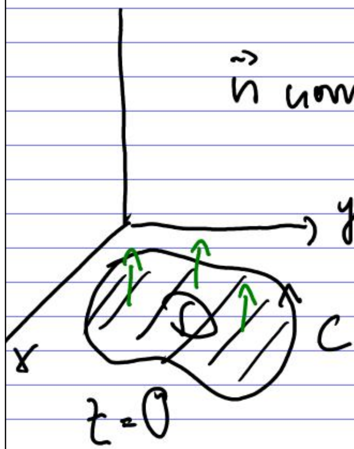
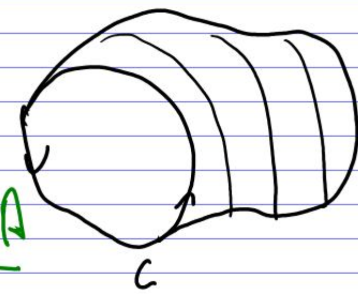
Panel 1

Stokes Theorem $\oint_C \vec{F}\, d\vec{r} = \iint_S \text{curl}(\vec{F})\cdot\vec{n}\, dS =$



$\vec{n}$ normal, $\vec{n} = \vec{k} = (0,0,1)$

$dS = \sqrt{f_x^2 + f_y^2 + 1}\, dA$

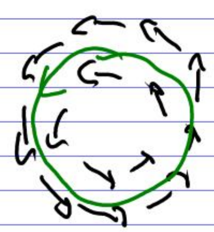


$= \iint_D \text{curl}(F)\cdot\vec{k}\, dA =$

Stokes thm reduces to Green's theorem

$= \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$

Panel 2

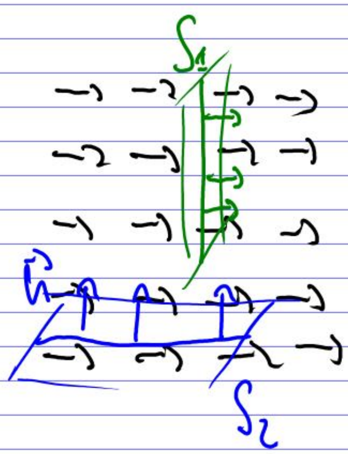


$\oint_C \vec{F}\, dr > 0$

$a \cdot b = \|a\|\|b\|\cos(\theta)$

Estimate Flux

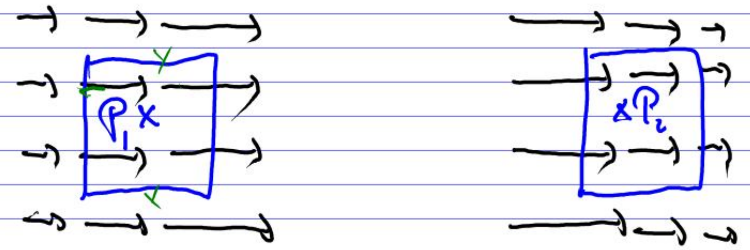
$\iint_{S_1} \vec{F}\cdot\vec{n}\, dS > 0$



$\iint_{S_2} \vec{F}\cdot\vec{n}\, dS = 0$

Panel 3

$F_1 = \langle x, 0, 0 \rangle \quad x > 0 \qquad F_2 = \langle \frac{1}{x}, 0, 0 \rangle$



$\text{div}(F) = \langle -\frac{1}{x^2}, 0, 0 \rangle$

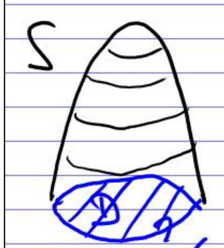
I want probe shaped like a small cube

$\iint_{S_1} \vec{F} \cdot \vec{n} \, dS > 0$ P_1 source $\qquad \iint_{S_2} \vec{F} \cdot \vec{n} \, dS < 0$ P_2 sink

$= \iiint \text{div}(F_1) \, dV > 0 \qquad = \iiint \text{div}(F_2) \, dV < 0$

Panel 4

Ex: $F = \langle 2z, 3x, 5y \rangle$, $S: z = 4 - x^2 - y^2, \; z \geq 0$



$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}(F) \cdot \vec{n} \, dS =$$

$n = \langle -f_x, -f_y, 1 \rangle = \langle 2x, 2y, 1 \rangle$

$$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2z & 3x & 5y \end{vmatrix} = \underset{M \; N \; P}{\langle 5, 2, 3 \rangle}$$

$$= \iint_{\text{disk, radius } 2} (2x \cdot 5 + 2y \cdot 2 + 1 \cdot 3) \, dA =$$

$$= \int_0^{2\pi} \int_0^2 (10 r \cos(t) + 4 r \sin(t) + 3) \, r \, dr \, dt = 12\pi$$

Panel 5

Green's, Stokes, Gauss' Thm

Green: $$\oint_C \vec{F}\, d\vec{r} = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$ C curve that bounds D in $\mathbb{R}^2$

Stokes $$\oint_C \vec{F}\, d\vec{r} = \iint_S \text{curl}(\vec{F}) \cdot \hat{n}\, dS$$ C curve that bounds S in $\mathbb{R}^3$

Gauss: $$\iint_S \vec{F} \cdot \hat{n}\, dS = \iiint_Q \text{div}(\vec{F})\, dV$$ S surface encloses Q in $\mathbb{R}^3$

Panel 6

b) $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = < \overset{M}{2xy^3 + x\cos(x)}, \overset{N}{3x^2y^2 - \sin(y)e^y} >$ and C is the closed curve given by the boundary of the square with corner points (-1,-1), (-1,1), (1, -1), and (1,1).

$$\oint_C \vec{F}\, d\vec{r} = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA = \iint_D 6xy^2 - 6xy^2\, dA = 0$$

c) $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x,y,z) = < x + z^2, 2y - \cos(xz), 3z >$ and S is given by $x^2 + y^2 + z^2 = 9$

$$\iint_S \vec{F}\, \hat{n}\, dS = \iiint_Q \text{div}(\vec{F})\, dV = \iiint_Q 1+2+3\, dV = 6\,\text{vol}(Q)$$
$$= 6 \cdot \frac{4}{3}\pi\, 3^3$$

d) $\int_C \vec{F}\, dr$ where $F(x,y,z) = < z^2, x^2, y^2 >$ and C is the boundary of the surface S given ~~by~~ $z = 1 - x - y$, restricted by the coordinate planes and oriented counter-clockwise.

$$\int_C \vec{F}\, d\vec{r} = \iint_S \text{curl}(\vec{F})\, \hat{n}\, dS$$

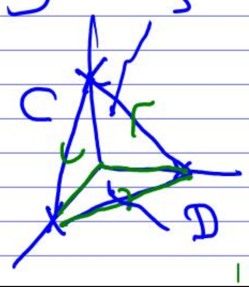
Panel 7

$$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ z^2 & x^2 & y^2 \end{vmatrix} = \langle 2y, 2z, 2x \rangle$$

$$\iint_S \text{curl}(F) \cdot \vec{n}\, dS = \iint_D \langle 2y, 2z, 2x \rangle \cdot \langle 1,1,1 \rangle\, dA =$$

$$= \iint_D 2(x+y+z)\, dA =$$

$$= \int_0^1 \int_0^{1-x} 2(x+y+z)\, dy\, dx = \checkmark$$



$z = 1 - x - y$