Panel 1

Integration Summary

Contour integrals

- function
  - $\int_\gamma f\,ds$, $ds = \sqrt{x'^2 + y'^2}\,dt$
  - $\int_\gamma f\,dx$ $dx = x'\,dt$
  - $\int_\gamma f\,dy$, $dy = y'\,dt$
- vector field $\int_\gamma \vec{F}\,d\vec{r}$
  - $\int_\gamma M\,dx + N\,dy$
  - $\vec{F}$ conserv. $f(b) - f(a)$
  - closed curve γ $\iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA$

Panel 2

② Evaluate the following integrals:

a) $\oint_\gamma \overbrace{(xy^2 + e^y)}\,dx + \overbrace{(x^2y + xe^y)}\,dy$, γ = unit circle

Green $= \iint 2xy + e^y - (2xy + e^y)\,dA$

b) $\int_{(-1,0)}^{(0,1)} (3x^2 + 2y)\,dx + (2x - 2y)\,dy = x^3 + 2xy - y^2 \Big|_{(-1,0)}^{(0,1)} = 0$

c) $\int_C 3x^2 - 7yx\,ds$, C line from $(0,1)$ to $(2,3)$

$ds = \sqrt{4+4}\,dt$ $r(t) = \langle 0,1\rangle + t\langle 2,2\rangle = \langle 2t, 1+2t\rangle$

d) $\int_\gamma \langle y, z, x\rangle \cdot d\vec{r}$, γ line from $(1,1,1)$ to $(2,3,4)$

e) $\oint \langle y, z, x\rangle\,d\vec{r}$, γ circle of radius 3

Panel 3

d) $\int_\gamma \langle y, z, x\rangle \cdot d\vec{r}$, γ line from $(1,1,1)$ to $(2,3,4)$

$f_x = y \Rightarrow f = xy + C(y,z)$

$f_y = x + C_y = z$

x don't drop out!

$\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ y & z & x \end{vmatrix} = \langle -1,$

$r(t) = \langle 1,1,1\rangle + t\langle 1,2,3\rangle = \langle 1+t, \underbrace{1+2t}, 1+3t\rangle$

$\int_\gamma \langle y, z, x\rangle\, d\vec{r} = \int_\gamma y\,dx + z\,dy + x\,dz =$

$= \int_0^1 (1+2t)\,dt + (1+3t)\,2\,dt + (1+t)\,3\,dt = \#$

Panel 4

e) $\oint \langle y, z, x\rangle\, d\vec{r}$, γ circle of radius 3 in (y,z)-plane

Not conservative $\Rightarrow$ not necessarily zero!

No antideriv. shortcut

circle is not closed in $\mathbb{R}^3$

Now: $r(t) = \langle 0, \underbrace{3\cos(t)}, 3\sin(t)\rangle$, $t \in [0, 2\pi]$

$\int y\,dx + z\,dy + x\,dz = \int \cancel{3\cos(t)\cdot 0} + 3\sin(t)(-3\sin(t))\,dt + 0$

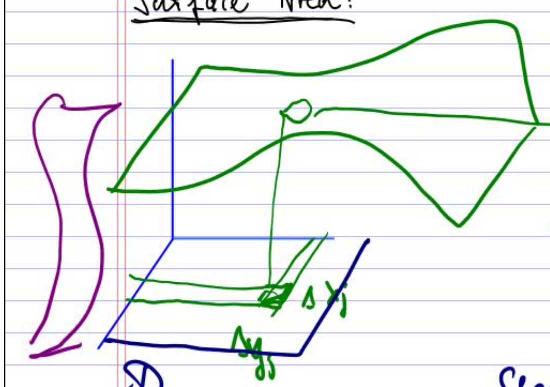
$= -9\int_0 \sin^2(t)\,dt = \#$

Panel 5

#7,4 on HW #25 would make good extra credit

Panel 6

Surface Area:



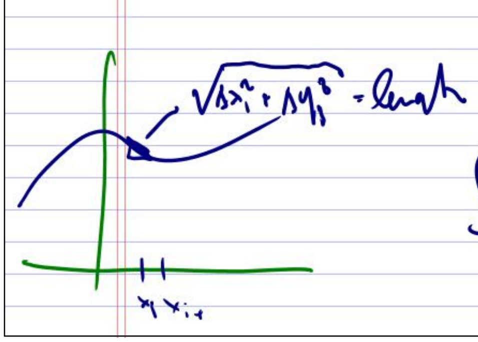
$z = f(x,y)$. Tangent plane

$z = f_x(x-x_0)+f_y(y-y_0)+z_0$

area( ) $= \sqrt{f_x^2+f_y^2+1}$

Surface area $\sum_{i,j=1}^{n}$ area( ) →

as $n \to \infty$ we get



$$S = \iint_D \sqrt{f_x^2+f_y^2+1}\,dA$$

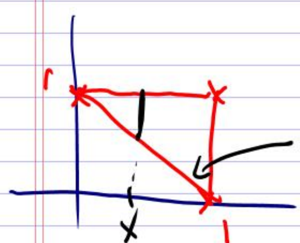
dA: $dx\,dy$ / $dy\,dx$ / $r\,dr\,d\theta$

Panel 7

Ex: Surface area of $z = x^2 + 2y$ above triangle $(0,1)$, $(1,0)$, and $(1,1)$

$$S = \iint_T \sqrt{4+4x^2+1}\, dA \qquad f_x = 2x,\quad f_y = 2$$

$$= \iint_T \sqrt{5+4x^2}\, dA = \int_0^1 \int_{1-x}^1 \sqrt{5+4x^2}\, dy\, dx =$$



$y = 1 - x$

$$= \int_0^1 \sqrt{5+4x^2}\,\big(1-(1-x)\big)\, dx = \int_0^1 x\sqrt{5+4x^2}\, dx = \frac{2}{3}\cdot\frac{1}{8}\left(5+4x^2\right)^{3/2}\Big|_0^1 \ \#$$

Panel 8

If S is given by $y = f(x,z)$

$$\Rightarrow S = \iint_D \sqrt{f_x^2 + f_z^2 + 1}\, dA \quad dA = \begin{cases} dx\, dz \\ dz\, dx \\ r\, dr\, d\theta \end{cases}$$

Surface of sphere? No can do because a sphere is not a function $z = f(x,y)$

But $1/2$ sphere works!

Panel 9

Find surface area of sphere, radius R, $(4\pi R^2)$

$z=\sqrt{R^2-x^2-y^2}$, is sphere upper half.

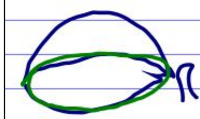
$f_x = \frac{1}{2}(R^2-x^2-y^2)^{-1/2}\cdot(-2x)$, $f_y = \frac{y}{(R^2-x^2-y^2)^{1/2}} \Rightarrow f_x^2+f_y^2+1 =$

$$\frac{x^2}{R^2-x^2-y^2}+\frac{y^2}{R^2-x^2-y^2}+1$$

$$\frac{R^2}{R^2-x^2-y^2}$$

$$S = 2\iint_{circle}\sqrt{f_x^2+f_y^2+1}\,dA =$$

$$2\iint_{circle}\sqrt{\frac{R^2}{R^2-x^2-y^2}}\,dA =$$



$$2R\iint_{circle}(R^2-x^2-y^2)^{-1/2}\,dA = 2R\int_0^{2\pi}\int_0^{R}(R^2-r^2)^{-1/2}\,r\,dr\,d\theta = 2R\,2\pi\left(-(R^2-r^2)^{1/2}\right)\Big|_0^R$$

$$= 4\pi R(-0+R) = \underline{\underline{4\pi R^2}}$$

Panel 10

Area of circle. πR^2

$$A = \int_{-R}^{R}\sqrt{R^2-x^2}\,dx \quad = HW$$

$x = R\sin(t)$

Length of circle $2\pi R$

$$C = \int_0^{2\pi}\sqrt{x'^2+y'^2}\,dt$$, $x=R\cos t$, $y=R\sin(t)$ ✓

polar coords

volume of sphere $\frac{4}{3}\pi R^3 = \iint_{circle}\sqrt{R^2-x^2-y^2}\,dA$

Panel 11

$y = f(x)$ a curve: $\vec{r}(t) = \langle t, f(t) \rangle$

$$\int ds = \int_a^b \sqrt{1 + f'^2}\, dx$$

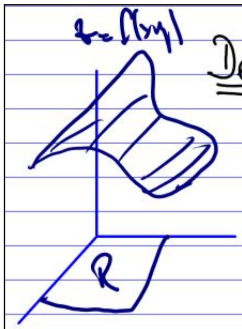
$$\int g(x,y)\, ds = \int g(x, f(x)) \sqrt{1 + f'^2}$$

$z = f(x,y)$ a surface

$$\iint dS = \iint \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$$\iint_R g(x,y,z)\, dS = \iint g(x,y,f(x,y)) \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

Panel 12



Def: Suppose surface S is defined by $z = f(x,y)$, $(x,y) \in R$. Then

$$\iint_S g(x,y,z)\, dS = \iint_R g(x,y, f(x,y)) \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$\iint_S g(x,y,z)\, dS$ is the integral of that portion of $w = g(x,y,z)$ over surface S where S is a surface defined on R

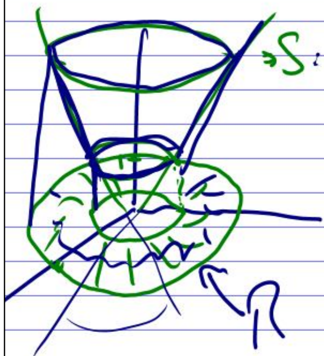
(no physical meaning)

Panel 13

Ex: Evaluate $\iint_S (x^2 z)\, dS$, S portion of $z^2 = x^2+y^2$ between $z=1$ and $z=4$.

$1 = x^2+y^2$

$(2)^2 = x^2+y^2$



$\Rightarrow S: \sqrt{x^2+y^2} = z$

$S: z = \sqrt{x^2+y^2} = f(x,y)$

$$\iint_S x^2 z\, dS = \iint_R x^2 \sqrt{x^2+y^2}\, \underbrace{\sqrt{f_x^2+f_y^2+1}}_{\sqrt{2}}\, dA$$

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}, \quad f_y = \dfrac{y}{\sqrt{x^2+y^2}} \qquad f_x^2+f_y^2+1 = \dfrac{x^2+y^2+x^2+y^2}{x^2+y^2} = 2$