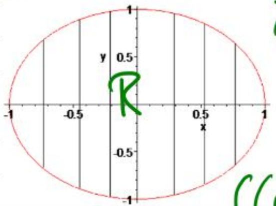
Panel 1

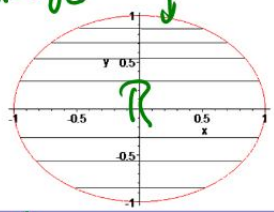
Name:

Quiz

① The pictures below show to different ways that a region R in the plane can be covered. Which picture corresponds to the integral $\iint_R f(x,y)\,dxdy$

$\iint_R f(x,y)\,dxdy$

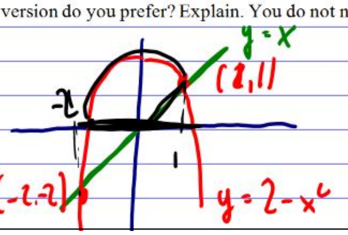




$\iint_R f(x,y)\,dydx$

② Suppose you want to evaluate $\iint_R f(x,y)\,dA$ where R is the region in the xy plane bounded by $y=0$, $y=2-x^2$, and $y=x$.

According to Fubini's theorem you could use either the iterated integral $\iint f(x,y)\,dxdy$ or $\iint f(x,y)\,dydx$ to evaluate the double integral. Which version do you prefer? Explain. You do not need to actually work out the integrals.



dy dx

y = 0 - logs

$2 - x^2 = x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

Panel 2

③ Evaluate the following integrals

a) $\int_0^1\int_0^2 xy^2\,dxdy$ $= 2/3$

b) $\int_0^2\int_{x^2}^{x}\left(x^2+2y\right)dydx$ $= -\frac{10}{15}$

$\int_2^3 x^2\,dx > 0$, $\int_3^2 x^2\,dx = -\int_2^3 x^2\,dx$

Panel 3

Tricks for Double-Integration

$\iint_R f(x,y)\,dx\,dy$ switch to $\iint_R f(x,y)\,dy\,dx$

Draw R & convert the bounds

Use geometry (volumes of known objects)

$\int_0^3\int_1^4 5\,dx\,dy = 45$

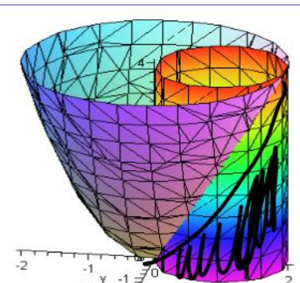
$x = r\cos\theta$

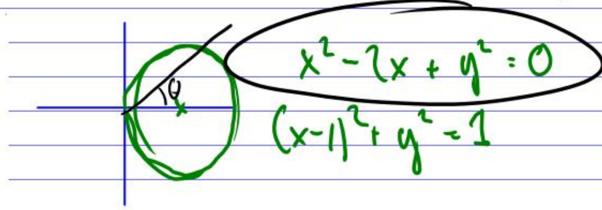
$y = r\sin\theta$

Polar Coordinates: $\iint_R f(x,y)\,dx\,dy = \iint_R f(r,\theta)\, r\,dr\,d\theta$

Panel 4

Ex: Volume under $z = x^2+y^2$, inside $x^2+y^2 = 2x$, above xy-plane





with(plots) :
implicitplot3d($\{z = x^2 + y^2, x^2 + y^2 = 2x\}$, $x = -2..2$, $y = -2..2$, $z = 0..4$);

$x = r\cos\theta$ $\quad (r\cos\theta)^2 - 2r\cos(\theta) + (r\sin\theta)^2 = 0$

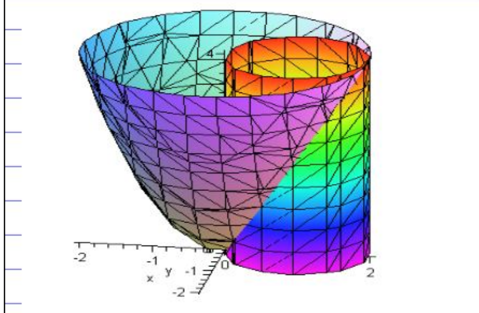
$y = r\sin\theta$ $\quad r^2 - 2r\cos(\theta) = 0$

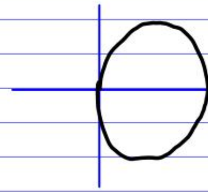
$r(r - 2\cos\theta) = 0 \quad r = 2\cos\theta$

$\iint_R x^2+y^2\,dA$ as $\theta \in [0, 2\pi]$, $r = [0, 2\cos\theta]$

Panel 5

Ex: Volume under $z = x^2+y^2$, inside $x^2+y^2 = 2x$, above xy-plane





$r = 2\cos\theta$

$$\iint_R x^2+y^2\,dA = \int_0^{2\pi}\int_0^{2\cos\theta} r^2\, r\,dr\,d\theta = \int_0^{2\pi}\frac{1}{4}(2\cos\theta)^4\,d\theta$$

$$= \underline{\underline{\frac{3\pi}{2}}}$$

Panel 6

Application of Integration

Suppose we have a lamina in the shape $D \subset \mathbb{R}^2$ that has a density function $\rho(x,y)$. Then

$$m = \iint_R \rho(x,y)\,dA$$

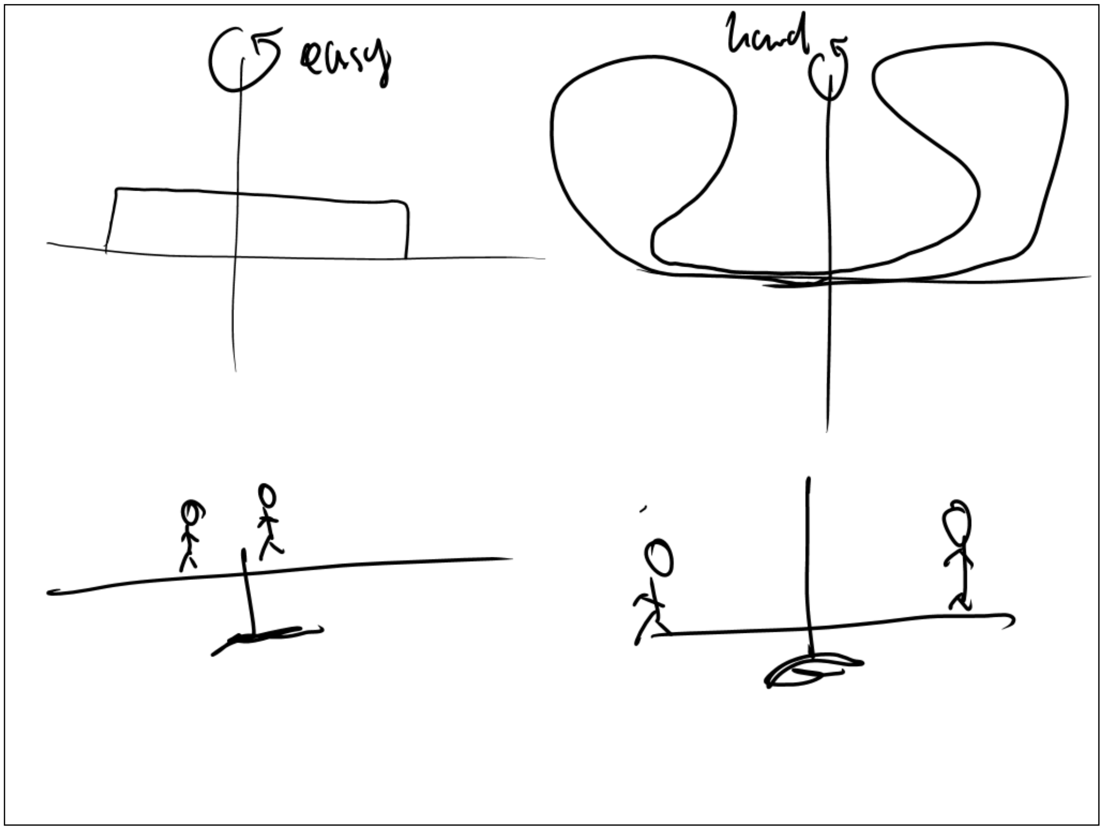
think of this as a "wedge"

$M_x = \iint_R y\,\rho(x,y)\,dA$ Moment about x-axis

$M_y = \iint_R x\,\rho(x,y)\,dA$ Moment about y-axis

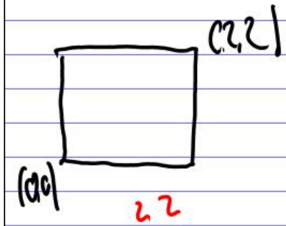
Finally, the center of mass is $\left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Panel 7



Panel 8

Find Center of Mass: first guess (1,1) then confirm



uniform density $\rho(x,y) = k$

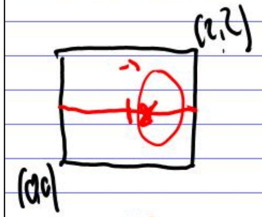
$$m = \int_0^2\int_0^2 k\,dx\,dy = 4k$$

$$M_x = \int_0^2\int_0^2 yk\,dx\,dy = 2\cdot\tfrac{1}{2}\cdot 4k = 4k \qquad M_y = \int_0^2\int_0^2 xk\,dx\,dy = 4k$$

$$\left(\frac{4k}{4k}, \frac{4k}{4k}\right) = (1,1) = (\bar{x},\bar{y}) \checkmark$$

Panel 9

Find Center of Mass: first guess then confirm



$\rho(x,y) = x$

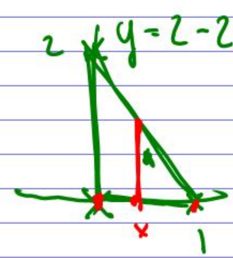
$m = \int_0^2\int_0^2 x \, dx\,dy = 2 \cdot \frac{1}{2} 2^2 = 4$

$M_x = \int_0^2\int_0^2 xy \, dx\,dy = 4$ $\qquad M_y = \int_0^2\int_0^2 y^2 \, dx\,dy = \frac{16}{3}$

$\bar{x} = \frac{M_y}{m} = \frac{16/3}{4} = \frac{4}{3}$ $\qquad \bar{y} = \frac{M_x}{m} = 1$

Panel 10

Example: Find center of mass of the triangular lamina (0,0), (1,0), (0,2) and density function $\rho(x,y) = 1+3x+y$



$m = \int_0^1\int_0^{2-2x} 1+3x+y \, dy\,dx = 8/3$

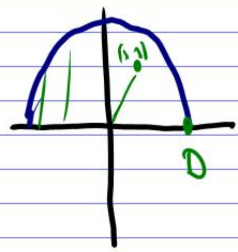
$M_x = \int_0^1\int_0^{2-2x} y(1+3x+y)\,dy\,dx$

$M_y = $ similar

Ex $(\bar{x}, \bar{y}) = \cancel{(\;,1)} = \left(\frac{3}{4}, \frac{11}{16}\right)$

Panel 11

Ex: Lamina density at any point on a semicircle is proportional to the distance from the center. Find $(\bar{x}, \bar{y})$, i.e center of gravity.



$$m = k \int_0^{\pi} \int_0^{r} \sqrt{x^2+y^2}\, dA \qquad r\,dr\,d\theta$$

Polar coords

$$M_x = k \iint y \sqrt{x^2+y^2}\, dA$$

$$M_y = k \iint x \cdot \sqrt{x^2+y^2}\, dA$$

Panel 12

Tripple Integration

Just like partial derivatives, integration can be extended to higher dimension without problems.

Ex: $\iiint_Q xyz \, dV$, Q a cube in 3D, side length 2

$[0,2]\times[0,2]\times[0,2]$ $\quad = \int_0^2 \int_0^2 \int_0^2 xyz \, dx\,dy\,dz =$

HW