Panel 1

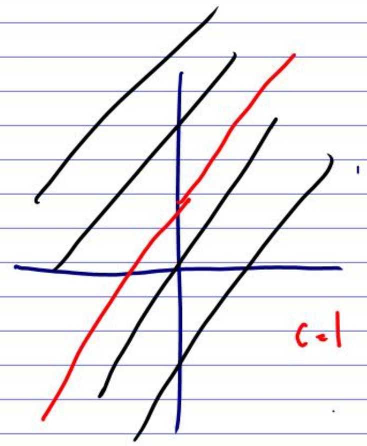
A level curve for $z = f(x,y)$ is a curve of constant height $c = f(x,y)$.

Ex. $f(x,y) = y - 2x$ Find level c at height 1:

$$y - 2x = 1$$

A collection of level curves is called contour plot, or topographical map.

$y - 2x = c$



Panel 2

To tackle limits in $\mathbb{R}^2$ (or $\mathbb{R}^3$)

① Try the obvious i.e. plug in, hope for the best

② Try different approach paths. Commonly used:

$x = 0$
$y = 0$
$x = y$
$x = y^2$, $y = x^2$, $x = y^3$, $y = x^3$, ...

if either answers are different, DNE

③ Try to prove that the limit is the common number found in step ②

Exam

Panel 3

Ex: $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+\cancel{y^2}}$

$x=0$, 0

$y=0$: 1

Ex: $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

$x=0: 0$

$y=0: 0$

$x=y$: $\frac{2x^3}{x^4+x^2} = \frac{2x^3}{x^2(x^2+1)} = 0$

$y=x^2$: $\frac{2x^4}{x^4+x^4} \Rightarrow 1$

Ex: $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$

Panel 4

Prove that $\lim_{(x,y)\to(0,0)} \frac{3x^3}{x^2+y^2} = 0$

For every $\varepsilon>0$ you need to find $\delta>0$ such that

if $\|(x,y)-(0,0)\| < \delta$ then $|f(x,y)-0| < \varepsilon$

$\sqrt{x^2+y^2} < \delta \Rightarrow \left|\frac{3x^3}{x^2+y^2}\right| < \varepsilon$

Say: $\frac{x^2}{x^2+y^2} \leq 1 \Rightarrow \frac{|x|x^2}{x^2+y^2} < |x| = \sqrt{x^2} \leq \sqrt{x^2+y^2}$

$\left|\frac{3x^3}{x^2+y^2}\right| < 3\sqrt{x^2+y^2}$

Panel 5

So we showed, for no appearant reason:

$$\left|\frac{3x^3}{x^2+y^2}\right| \le 3\sqrt{x^2+y^2}$$

But with this fact established, we can now do the real proof. It goes like this:

take any $\varepsilon > 0$, I pick $\delta = \frac{1}{3}\varepsilon$. Then, if

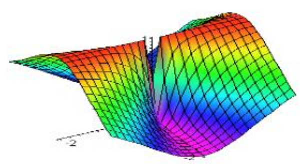
$$\|x,y\| = \sqrt{x^2+y^2} < \delta = \frac{1}{3}\varepsilon \Rightarrow 3\sqrt{x^2+y^2} < \varepsilon$$

But $\left|\frac{3x^3}{x^2+y^2}\right| < \sqrt{x^2+y^2} < \varepsilon \Rightarrow |f(x,y)| < \varepsilon$.

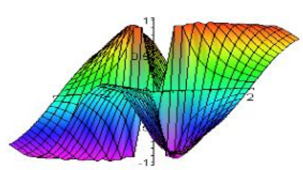
That's it! #

Panel 6

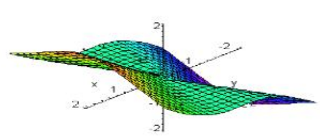
$$f(x,y) = \frac{x^2}{x^2+y^2}$$ ✓



$$f(x,y) = \frac{2x^2y}{x^4+y^2}$$ ✓



$$f(x,y) = \frac{x^3}{x^2+y^2}$$

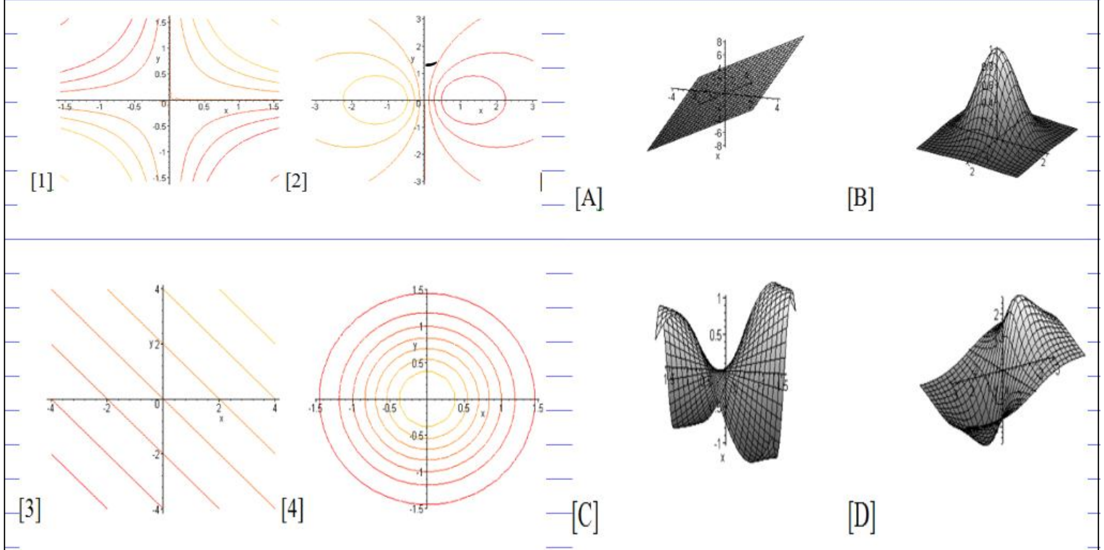


Panel 7

Quiz

Name: ______

① Match the contour plots to the graphs:



Panel 8

② Find the limits or show that it does not exist:

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy+1}{x^2+y^2+1}$

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6+y^2}$

Panel 9

Friday 2pm-5pm: Trends and Advances in Computing: Systems, Paradigms, and Architecture

2-2:45: The Android OS (Bert Wachsmuth)

3-4: Quantum Computing (Tom MarLowe)

4-5: Atomic Scale Materials for Computers of the Future (Alper Sahiner)

Panel 10

Continuity

As usual, continuity is just

Def. $f(x,y)$ is continuous at (x_0,y_0) if

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$$

(1) lim
(2) $f(x_0,y_0)$
(3): (1) = (2)

Ex: $f(x,y) = \begin{cases} \frac{3x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ \cancel{2}\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

Know: $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = 0$, $f(0,0) = 2 \Rightarrow$ No

YES

Panel 11

Derivatives: 2 vars ⇒ 2 deriv.

If $f(x,y)$ is a function of 2 variables, define

$$\lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} = \frac{\partial f}{\partial x} = f_x \quad \text{partial w.r.t. } x$$

$$\lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h} = \frac{\partial f}{\partial y} = f_y \quad \text{partial w.r.t. } y$$

Panel 12

Ex: Find f_x if $f(x,y) = x^2y + y^2$

$$\lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} = \lim_{h\to 0} \frac{(x+h)^2y + \cancel{y^2} - x^2y - \cancel{y^2}}{h}$$

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2)y - x^2y}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{x^2y} + 2xyh + h^2y - \cancel{x^2y}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2xy+hy)}{\cancel{h}}$$

$$= 2xy$$

$f_y = x^2 + 2y$ Really: f_x: Keep y const.

Panel 13

Ex: $f(x,y) = x^3 + x^2y^3 - 2y^2$. Find

$f_x(2,1)$: $f_x(x,y) = \frac{\partial f}{\partial x} = 3x^2 + 2xy^3$

$f_x(2,1) = 3\,2^2 + 2\cdot 2\cdot 1^3 = \underline{\underline{16}}$

$f_y(2,1)$: $f_y(x,y) = 3x^2y^2 - 4y$

$= 3\cdot 2^2\cdot 1^2 - 4\cdot 1 = \underline{\underline{8}}$

Panel 14

3D Example: $f(x,y,z) = xz\,e^{x^2+y^2}$. Find

$f_x(x,y,z) = z\,e^{x^2+y^2} + xz\,2x\,e^{x^2+y^2}$

$f_y(x,y,z) = xz\,e^{x^2+y^2}\cdot 2y$

$f_z(x,y,z) = x\,e^{x^2+y^2}$

Panel 15

Of course we can also take higher-order partial derivatives. Let $f(x,y)$ be a function

1st order: $f_x(x,y)$ $\quad f_y(x,y)$

2nd order: f_{xx} $\quad f_{xy}$ $\quad f_{yx}$ $\quad f_{yy}$

3rd order: f_{xxx} $\quad f_{xxy}$ $\quad f_{xyx}$ $\quad f_{xyy}$ $\quad f_{yxx}$ $\quad f_{yxy}$ $\quad f_{yyx}$ $\quad f_{yyy}$

Panel 16

Ex: $f(x,y) = x^3 + x^2y^3 - 2y^2$

$f_x(x,y) = 3x^2 + 2xy^3$

$f_y(x,y) = 3x^2y^2 - 4y$

$f_{xx}(x,y) = (f_x)_x = 6x + 2y^3$

$f_{xy}(x,y) = (f_x)_y = 6xy^2$

$f_{yx}(x,y) = (f_y)_x = 6xy^2$

$f_{yy}(x,y) = (f_y)_y = 6x^2y - 4$