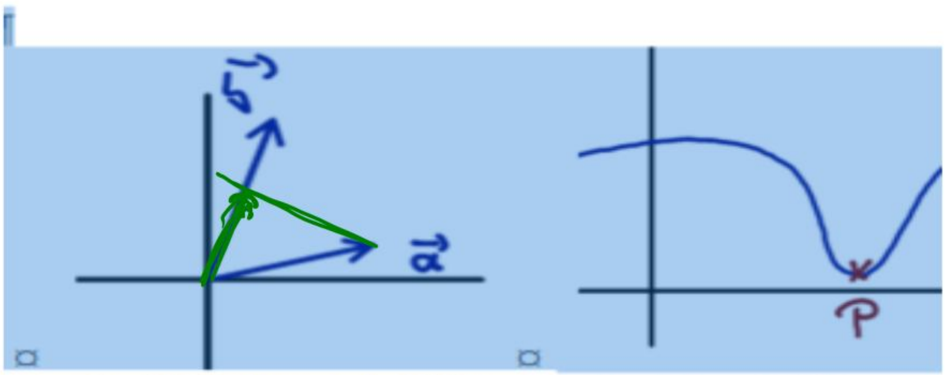
Panel 1



c) Sketch $proj_{\vec{b}}(\vec{a})$

d) Sketch $\vec{T}$ (tang
and $\vec{N}$ (normal)

, and $\vec{w} = < 2,0,-3 >$

Panel 2

Functions of Several Variables

Know. $f: \mathbb{R} \to \mathbb{R}$ e.g

$\mathbb{R} \to \mathbb{R}^2$ e.g.

$\mathbb{R} \to \mathbb{R}^3$ e.g. space curves

$$\left.\begin{array}{l} f: \mathbb{R}^2 \to \mathbb{R} \\ \mathbb{R}^3 \to \mathbb{R} \end{array}\right\} \text{next}$$

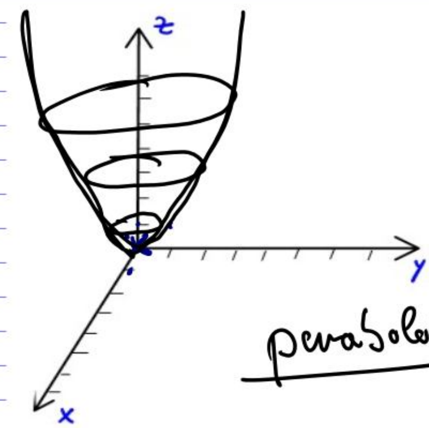
Panel 3

Def: A function of 2 variables is a rule that assigns to every pair (x,y) in a set $D \subset \mathbb{R}^2$ exactly one number $z = f(x,y)$

↑ domain

Ex: $f(x,y) = x^2 + y^2 = z$

(x,y)	$z = f(x,y)$
$(0,0)$	0
$(1,0)$	1
$(-1,0)$	1
$(0,\pm 1)$	1
$(\pm 1,\pm 1)$	2



paraboloid

What if z = constant

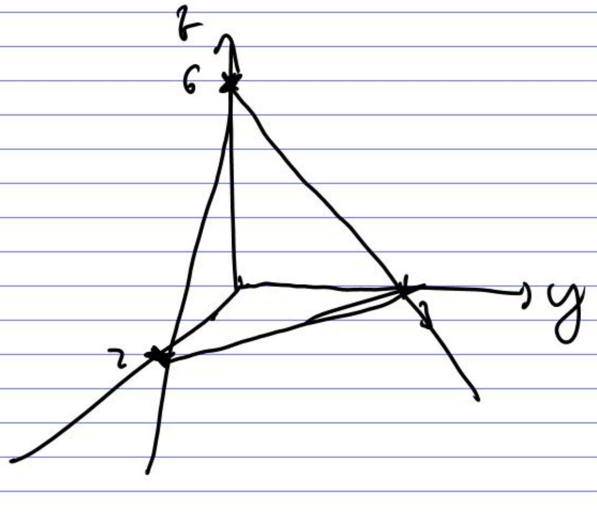
e.g. $z = 4 = x^2 + y^2$

Panel 4

Ex: $f(x,y) = 6 - 3x - 2y = z$

What is $z = 6 - 3x - 2y$

$3x + 2y + z = 6$ plane



Panel 5

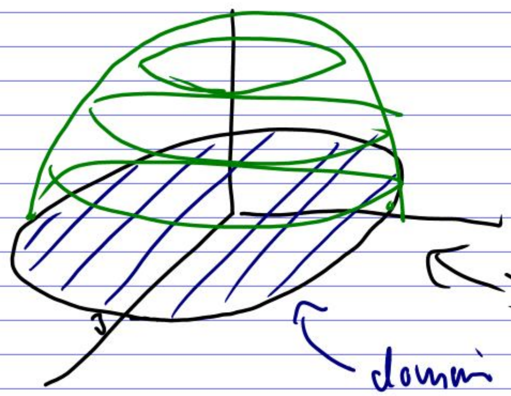
Ex: $f(x,y) = \sqrt{9-x^2-y^2} = z$

Domain. $9-x^2-y^2 \geq 0$

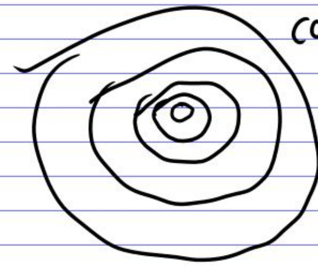
$9 \geq x^2+y^2$ $\quad (9 = x^2+y^2)$

$z^2 = 9-x^2-y^2$

$x^2+y^2+z^2=9$



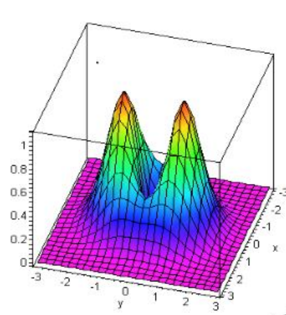
domain

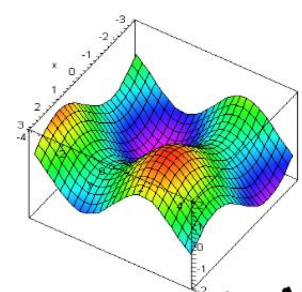


contour plot

Panel 6

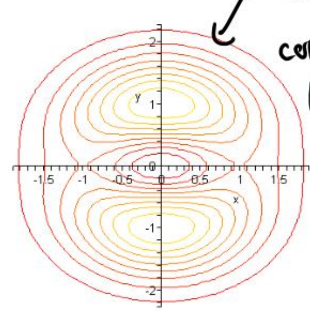
$f(x,y) = (x^2+3y^2)e^{-x^2-y^2}$ $\quad$ $f(x,y) = \sin(x)+\sin(y)$

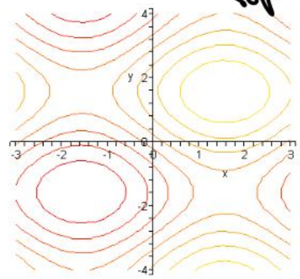




topographical maps

lines of constant height





contour plot

Panel 7

Of course I used Maple to generate these plots

```
> plot3d((x^2+3*y^2)*exp(-x^2-y^2),x=-3..3,y=-4..4);
> plot3d(sin(x)+sin(y),x=-3..3,y=-4..4);
> with(plots);
> contourplot((x^2+3*y^2)*exp(-x^2-y^2),x=-3..3,y=-4..4);
> contourplot(sin(x)+sin(y),x=-3..3,y=-4..4);
>
```

Wolfram Alpha: sin(x)+cos(y)

or

contourplot sin(x)+cos(y)

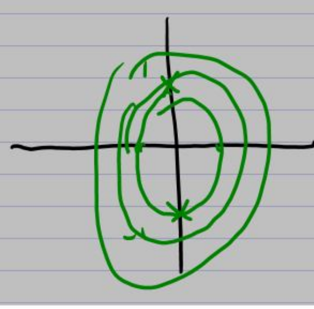
Panel 8

Ex: Level curves of $h(x,y) = 4x^2+y^2$

$4x^2+y^2=1$

$4x^2+y^2=2$

$4x^2+y^2=c$

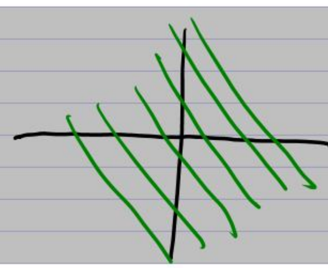


Ex: Level curves for $f(x,y) = 6-3x-2y$

$6-3x-2y=c$

$2y = 6-c-3x$

$y = \frac{1}{2}(6-c-3x)$



Panel 9

How about: $f:\mathbb{R}^3\to\mathbb{R}$, e.g.

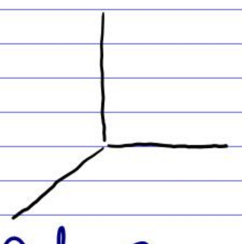
$$f(x,y,z)=x^2+y^2+z^2$$

$w=x^2+y^2+z^2$

Graph in 4D $\Rightarrow$ pass

Panel 10

Limits: The limit of $f(x,y)$ as (x,y) approaches (x_0,y_0) is L is written as



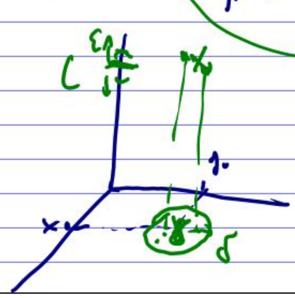
$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

Def: Given any $\varepsilon>0$ there is $\delta>0$ such that

$|f(x,y)-L|<\varepsilon$ if $\|(x,y)-(x_0,y_0)\|<\delta$

$\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$

for each



$f:\mathbb{R}^2\to\mathbb{R}$

Panel 11

In $\mathbb{R}$: $\lim_{x \to x_0} f(x)$

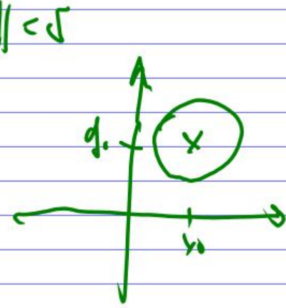
$|x - x_0| < \delta$

2 ways to get close to x_0: left, right

In $\mathbb{R}^2$: $\lim_{(x,y) \to (x_0,y_0)} f(x,y)$

$\|(x,y) - (x_0,y_0)\| < \delta$

How many ways to get close to (x_0,y_0): infinitely many!



Limits are hard!!!

Panel 12

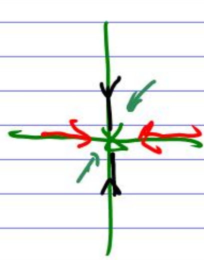
Hints for finding limits in $\mathbb{R}^2$:

$\Rightarrow$ if C_1 is a path to (x_0,y_0) and $f(x,y) \to L_1$ on C_1

$\Rightarrow$ if C_2 is a path to (x_0,y_0) and $f(x,y) \to L_2$ on C_2

If $L_1 \neq L_2$, the limit $(x_0,y_0) \to (x,y)$ Does Not exist

Ex: $\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist



$y = 0,\ x \to 0$: $\lim_{x \to 0} \frac{x^2 - 0}{x^2 + 0} = 1$

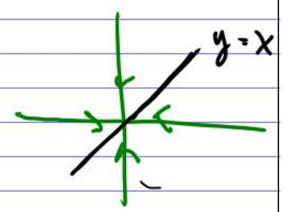
$x = 0,\ y \to 0$: $\lim_{y \to 0} \frac{-y^2}{+y^2} = -1$

differ so limit d.n.e.

Panel 13

Ex: $\lim\limits_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{1}{2}$

more work!



$\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \frac{0}{0}$

$y=0,\ x\to 0:\ \lim\limits_{x\to 0} \frac{0}{x^2} = 0$

$x=0,\ y\to 0:\ \lim\limits_{y\to 0} \frac{0}{y^2} = 0$

done

$x=y,\ x\to 0:\ \lim\limits_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$

Panel 14

Ex: Find $\lim\limits_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ if it exists (More work)

$x=0,\ y\to 0:\ \lim\limits_{y\to 0} \frac{0}{y^4} = 0$

Done. done.

$y=0,\ x\to 0:\ \lim\limits_{x\to 0} \frac{0}{x^2} = 0$

l'Hospital?

$x=y,\ x\to 0:\ \lim\limits_{x\to 0} \frac{x^3}{x^2+x^4} = \lim\limits_{x\to 0} \frac{3x^2}{2x+4x^3} = \frac{6x}{2+12x^2} \to 0$

$x=y^2,\ y\to 0:\ \lim\limits_{y\to 0} \frac{y^4}{2y^4} = \frac{1}{2}$

$y=x^2,\ x\to 0 \Rightarrow 0$

Panel 15

Ex: $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$

$x=0,\ y\to 0:\ 0$

$y=0,\ x\to 0:\ 0$

$x=y:\ \lim_{x\to 0} \frac{3x^3}{2x^2} = 0$

$x=y^2:\ \lim \frac{3y^5}{y^4+y^2} = 0$

$y=x^2:\ \lim \frac{3x^4}{x^2+x^4} = 0$

Give up: maybe limit exists???
(Could only be 0)

Panel 16

Prove that $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} = 0$

Given $\varepsilon>0$ there is $\delta>0$ s.t.

if $\|(x,y)-(0,0)\|<\delta$ then $|f(x,y)-0|<\varepsilon$

if $\sqrt{x^2+y^2}<\delta$ then $\left|\frac{3x^2y}{x^2+y^2}\right|<\varepsilon$

Know: $x^2 \le x^2+y^2 \Rightarrow \frac{x^2}{x^2+y^2} \le 1$

$\Rightarrow \frac{3|y|x^2}{x^2+y^2} \le 3|y| = 3\sqrt{y^2} \le 3\sqrt{x^2+y^2}$

Now what

Thus: $\sqrt{x^2+y^2} \ge \frac{1}{3}\frac{3|y|x^2}{x^2+y^2} = \frac{1}{3} f(x,y)$?