Panel 1

Motion in Space

Suppose $\vec{r}(t)$ represents the motion of a particle in space over time (t)

$\vec{r}(t)$ - path (motion)

$\vec{v} = \vec{r}\,'(t)$ = velocity , $s = \|r'\|$ - speed

$\vec{a} = \vec{r}\,''(t)$ = acceleration , $\vec{a} = \vec{v}\,'$

Panel 2

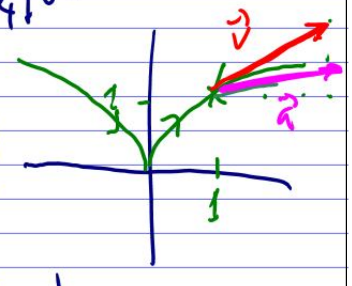
Ex: Suppose the path of a particle at time t is $\vec{r}(t) = \langle \underset{x}{t^3}, \underset{y}{t^2} \rangle$. Find velocity, speed, and acceleration when t=1. Illustrate.

$\vec{v}(t) = r' = \langle 3t^2, 2t \rangle$

$\vec{v}(1) = \langle 3, 2 \rangle$

$s = \sqrt{9+4} = \sqrt{13}$

$= \sqrt{9t^4 + 4t^2}$



$a'(t) = \langle 6t, 2 \rangle$

$\vec{a}(1) = \langle 6, 2 \rangle$

$x = t^3$, $y = t^2$

$\sqrt[3]{x} = t$, $y = x^{2/3}$ $\left(y' = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3} \frac{1}{\sqrt[3]{x}} \right)$

$\vec{r}(1) = \langle 1, 1 \rangle$

Panel 3

Ex: A particle starts at P(1, 0, 0) with initial velocity $\langle 1, -1, 1\rangle$. The acceleration is $\vec{a}(t) = \langle 4t, 6t, 1\rangle$. Find velocity, speed, and position.

$\vec{a} = \langle 4t, 6t, 1\rangle$

$\rightarrow \vec{v} = \int \vec{a}\, dt = \langle 2t^2, 3t^2, t\rangle + C$

$v(0) = \langle 1, -1, 1\rangle$ C

$\vec{r}(t) = \langle \frac{2}{3}t^3, t^3, \frac{1}{2}t^2\rangle + \langle t, -t, t\rangle + D$

$\vec{r}(0) = \langle 1, 0, 0\rangle$

$\vec{r}(t) = \langle \frac{2}{3}t^3, t^3, \frac{1}{2}t^2\rangle + \langle t, -t, t\rangle + \langle 1, 0, 0\rangle$

Panel 4

Ex: An object with mass m moves in a circle with constant angular speed ω. Find the force acting on the object and illustrate.

$\vec{r}(t) = \langle \cos(\omega t), \sin(\omega t)\rangle$ $\quad r = \langle \omega\cos(t), \omega\sin(t)\rangle$

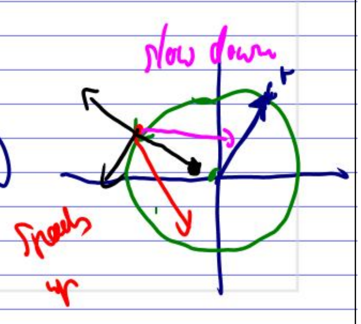
$\vec{r}'(t) = \langle -\omega\sin(\omega t), \omega\cos(\omega t)\rangle$ $\quad s = \omega$

$s = \cancel{1}\ \omega$

$\vec{F} = m\vec{a} = m\langle -\omega^2\cos(\omega t), -\omega^2\sin(\omega t)\rangle$

$= -\omega^2 m\vec{r}$

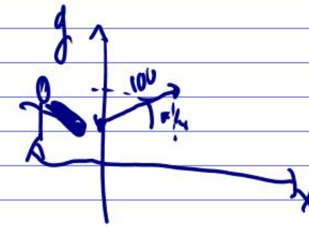
$\vec{F} \sim -\vec{r}$!!!



Panel 5

Application of Motion

A baseball is hit 3 feet above ground at 100 feet per second and at an angle of Pi/4 with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10-foot high fence located 300 feet from home base?



$\vec{a} = \langle 0, -g \rangle$ $\qquad g$ = gravity = 9.98 m/sec²

$\vec{v} = \langle c_1, -gt + c_2 \rangle$, $\vec{v}(0) = \langle 100\cdot\cos(\frac{\pi}{4}), 100\cdot\sin(\frac{\pi}{4}) \rangle$

$c = \frac{1}{\sqrt{2}}\langle 100, 100 \rangle$

$\vec{v} = \langle \frac{1}{\sqrt{2}}100, -gt + \frac{1}{\sqrt{2}}100 \rangle$

$\vec{r}(t) = \langle \frac{1}{\sqrt{2}}100t + \overset{\to 0}{\cancel{d_1}}, -\frac{1}{2}gt^2 + \frac{100}{\sqrt{2}}t + \overset{\to 3}{\cancel{d_2}} \rangle$, $r(0) = \langle 0, 3 \rangle$

Panel 6

$\vec{r}(t) = \langle \frac{100}{\sqrt{2}}t, -\frac{1}{2}gt^2 + \frac{100}{\sqrt{2}}t + 3 \rangle$

Max height. $\quad h(t) = -\frac{1}{2}gt^2 + \frac{100}{\sqrt{2}}t + 3$ is max:

$h'(t) = -gt + \frac{100}{\sqrt{2}} = 0 \Rightarrow t_0 = \frac{100}{\sqrt{2}\cdot g}$

Max height is $h(t_0)$

Home run: How high at 300 m distance?

$\frac{100}{\sqrt{2}}t = 300 \Rightarrow t_1 = \frac{300}{100}\sqrt{2} = 3\sqrt{2}$

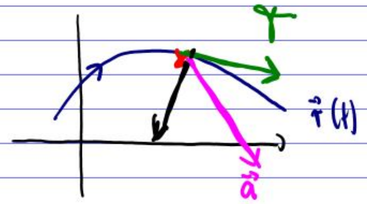
Height is $h(t_1) > 10$ Home run

< 10 Nada

Finish it: HW

Panel 7

Tangential and Normal Components of Acceleration



Acceleration has two components:

a) tangential in dir of T

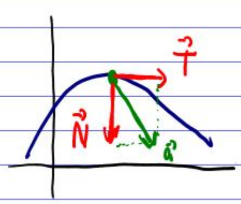
b) normal in dir. of $\vec{N}$

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

a_T = tangential comp. speeds up or slows down particle

a_N = normal comp. changes dir. of particle.

Panel 8



$$\vec{a} = a_t \cdot \vec{T} + a_N \vec{N}$$

$$T = \frac{r'}{\|r'\|} = \frac{\vec{v}}{s} \Rightarrow \vec{v} = s\vec{T}$$

$$\frac{d}{dt}\vec{v} = \vec{a} = s' T + s T'$$

$$\vec{N} = \frac{T'}{\|T'\|} \Rightarrow T' = N \|T'\| , \quad \kappa = \frac{\|T'\|}{\|r'\|} = \frac{\|T'\|}{s}$$

$$T' = N s \kappa$$

$$\vec{a} = s' \vec{T} + s^2 \kappa \vec{N}$$

Thus

$$a_T = s' \qquad a_N = s^2 \kappa$$

Panel 9

$\vec{a} = a_T \vec{T} + a_N \vec{N}$ with

$a_T = s'$ and $a_N = s^2 \chi$

$\chi = \frac{\|r' \times r''\|}{\|r'\|^3} = \frac{\|r' \times r''\|}{s^3}$ $\Rightarrow \underline{a_N} = \cancel{s^2} \cdot \frac{\|r' \times r''\|}{\cancel{s^3}} = \frac{\|r' \times r''\|}{s}$

$\vec{a} = s' T + s^2 \chi N \quad \Big| \cdot v = sT$

$\vec{a} \cdot \vec{v} = s' T \cdot (sT) + s^2 \chi N \cdot (sT) =$

$= s's \underbrace{T \cdot T}_{\|T\|^2 = 1} + s^3 \chi \cancel{N \cdot T}^{0}$ $\qquad a_T = \frac{\vec{a} \cdot \vec{v}}{s}$

$\vec{a} \cdot \vec{v} = s' \cdot s$

Panel 10

<u>Theorem:</u> $\vec{a} = a_T \vec{T} + a_N \vec{N}$ where

tang. component $a_T = \frac{\vec{a} \cdot \vec{v}}{s}$ $\qquad$ note: $a_N \geq 0$

normal component $a_N = \frac{\|\vec{a} \times \vec{v}\|}{s}$

<u>Ex:</u> $r(t) = \langle t^2, t^2, t^3 \rangle$ - find a_T and a_N

$\vec{v} = r'(t) = \langle 2t, 2t, 3t^2 \rangle \Rightarrow s = \sqrt{4t^2 + 4t^2 + 9t^4}$

$\vec{a} = r''(t) = \langle 2, 2, 6t \rangle$ $\qquad a \cdot v = 4t + 4t + 18t^3$

$\underline{a_T} = \frac{8t + 18t^3}{\sqrt{8t^2 + 9t^4}} = \underline{\frac{8 + 18t^2}{\sqrt{8 + 9t^2}}}$ $\qquad a_N = \frac{\|\vec{v} \times \vec{a}\|}{\sqrt{8t^2 + 9t^4}}$

Panel 11

Quiz 4

Suppose $\vec{r}(t)=\langle t^2,2,t\rangle$ is a vector-valued function (aka space curve), representing the position of a particle. Find the following:

HW

1. The velocity at $P(0,2,0)$
2. The speed at $P(0,2,0)$
3. The acceleration at $P(0,2,0)$
4. The unit tangent $\vec{T}(t)$ at $P(0,2,0)$
5. The unit normal vector $\vec{N}(t)$ at $P(0,2,0)$
6. The bi-normal vector $\vec{B}(t)$ at $P(0,2,0)$
7. The curvature k at $P(0,2,0)$
8. The tangential component of the acceleration a_T at $P(0,2,0)$
9. The normal component of the acceleration a_N at $P(0,2,0)$
10. The osculating plane at $P(0,2,0)$
11. The osculating circle at $P(0,2,0)$

Panel 12

Functions of Several Variables

Know: $f:\mathbb{R}\to\mathbb{R}$ e.g. $y=x^2$

$r:\mathbb{R}\to\mathbb{R}^2$ e.g. $r(t)=\langle t,t^3\rangle$

$r:\mathbb{R}\to\mathbb{R}^3$ e.g. $r(t)=\langle t, t\cos(t), t\sin(t)\rangle$

Next: $f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)=xy$

$f:\mathbb{R}^3\to\mathbb{R}$ $f(x,y,z)=xz+yx+z^2$

Last $f:\mathbb{R}^2\to\mathbb{R}^2$